

IOWA STATE UNIVERSITY

Digital Repository

Retrospective Theses and Dissertations

Iowa State University Capstones, Theses and
Dissertations

1968

Digital stability analysis of power generating units

Joseph Donald Musil
Iowa State University

Follow this and additional works at: <https://lib.dr.iastate.edu/rtd>

 Part of the [Electrical and Electronics Commons](#), and the [Oil, Gas, and Energy Commons](#)

Recommended Citation

Musil, Joseph Donald, "Digital stability analysis of power generating units " (1968). *Retrospective Theses and Dissertations*. 3501.
<https://lib.dr.iastate.edu/rtd/3501>

This Dissertation is brought to you for free and open access by the Iowa State University Capstones, Theses and Dissertations at Iowa State University Digital Repository. It has been accepted for inclusion in Retrospective Theses and Dissertations by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.

**This dissertation has been
microfilmed exactly as received**

69-4268

**MUSIL, Joseph Donald, 1920-
DIGITAL STABILITY ANALYSIS OF POWER
GENERATING UNITS.**

**Iowa State University, Ph.D., 1968
Engineering, electrical**

University Microfilms, Inc., Ann Arbor, Michigan

DIGITAL STABILITY ANALYSIS OF POWER GENERATING UNITS

by

Joseph Donald Musil

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Ames, Iowa

1968

TABLE OF CONTENTS

	Page
I. INTRODUCTION	1
II. REVIEW OF LITERATURE	3
III. SYSTEM STABILITY BY EIGENVALUES	7
A. Method of Analysis	7
B. Formulation of the Equation	8
C. Eigenvalue Routine	8
D. Extent of the Investigation	9
IV. SYNCHRONOUS GENERATOR	11
A. Operating Matrix Equations	14
B. Reduced System A-constants	14
C. Feedback Control Scheme	17
D. State-space Equations	18
E. Damping	21
F. Saturation	21
G. Tie-line Reactance	22
V. EXCITATION SYSTEM	25
A. Type 1 System	25
B. Power Damping Signal	32
C. Saturation	33
D. State-space Equations	35

VI.	GOVERNOR SYSTEM	37
	A. Single-shaft Governor Representation	37
	B. State-space Equations	37
VII.	TIE-LINE CONTROLLER	46
	A. Tie-line Controller Representation	46
	B. State-space Equations	46
VIII.	MULTI-MACHINE PROBLEM	49
	A. n-Machine Analysis	49
	B. Two-Machine System Equations	54
IX.	REMARKS AND CONCLUSIONS	55
	A. Machine Variables and A-constants	55
	B. Operation with Voltage Regulator	55
	C. Operation with Voltage Regulator and Governor	55
	D. General Observations	56
X.	LITERATURE CITED	87
XI.	ACKNOWLEDGMENTS	91
XII.	APPENDIX A, LIST OF PRINCIPAL SYMBOLS	92
XIII.	APPENDIX B, SYSTEM EQUATIONS AND A-CONSTANTS OF THE SYNCHRONOUS GENERATOR	96
	A. General Non-linear System Equations	96
	B. A-constants	98
XIV.	APPENDIX C, PROGRAMS FOR A-MATRICES AND EIGENVALUES FROM COMPONENT DATA	100

I. INTRODUCTION

The purpose of this investigation is to study the dynamic stability of a typically large (700 to 800 mva) synchronous generator unit with its automatic voltage regulator and governor when connected to an infinite bus through a typically long transmission line. Sets of higher-order differential equations describing the dynamical behavior of the various elements are formulated by state-space methods into sets of first-order equations. In matrix algebra the equations take the form $\dot{\underline{x}} = \underline{A} \underline{x} + \underline{D} \underline{u}$ where \underline{A} and \underline{D} are the matrices of constant coefficients, \underline{u} is the forcing function input vector and \underline{x} is the vector representing the state of the system variables. Stability is determined by digital methods in this investigation by insuring that the real part of the eigenvalues of the A-matrix at various states remains negative.

The voltage at the infinite bus is held at 1.0 per-unit (pu). The generator terminal voltage is iterated to the value needed to supply various power outputs and power factors of operation at the generator terminals. The variations of tie-line reactances and system damping are investigated.

The excitation control system is of a currently standardized type. A derivative power damping signal is used. The effect of varying the voltage regulator parameters and the addition of the governor system on stability are investigated.

Addition of a tie-line control signal to the governor and the extension of the method of investigation to a multi-machine system are discussed briefly.

II. REVIEW OF LITERATURE

For many years the network analyzer was the tool used to compute the steady-state as well as transient stability by establishing new levels of operation in system load flow. These basic concepts, given in Kimbark (19), Crary (9), and by Westinghouse (33) produce conservative results but give a sound background on which to add the refinements present-day computers permit.

Parks (24,25) established the direct and quadrature component presentation of the synchronous machine. Kilgore (18) and Doherty and Nickle (10) by calculation and testing, clearly defined and established the accuracy of the reactances of the machine. Rankin (26,27) categorized approaches on per-unit notation. These have helped form the basis for a systematized approach enjoyed in stability studies today.

Stability studies limited by the computation facilities available, were soon enlarged upon. Concordia's work (7) in 1944, studying the steady-state stability of the synchronous machine with the voltage regulator, is a classic reference. Heffron and Phillips (14) in 1952 undertook the study of operation in under-excited regions, made possible by the use of faster, rotating voltage regulators. Improvements in excitation control systems has led to lower short circuit ratio generators with more mega-volt-ampere capacity

from a given frame size. The excitation control systems which have been developed have been standardized (16) for system stability studies.

Trends mentioned above could only proceed as fast as computer facilities and programs evidenced stability. The November, 1964 Northeast power failure (13) caused new interest and concern in assaying the correctness of programs with respect to interconnected systems. Fortunately computer facilities were available.

Olive (23) in his program in 1965 gave one of the first papers on the problem of detailed programming of transient and steady-state stability permitting 500 machines, 1000 busses, and 1500 lines. Young and Webler (34) and their similar program permitted 250 machines, 1000 busses, and 2000 lines. Both programs contained the machine, control, and system representations deemed needful for these studies. Steady-state stability checks in these programs were accomplished by a step-input perturbation followed by a real-time run of five seconds or more to insure return to stable operation. Certain simplifications in transient stability programming were permissible for steady-state checks but the method of multi-step, swing-curve time solutions along with intermediate iterative load-flow adjustments are fundamental to this method. Many non-linearities can be accommodated. This method is costly but still presently necessary.

A large scale study of the current type in the paper by Lokay and Bolger (22) incorporates transient saliency, saturation, machine and system damping, speed governor system and excitation system with a 660 mw turbine-generator set. Another informative study by Ewart, Landgren, Temoshok, and Walkey (11) investigates a 532 mw cross-compound unit. Damper winding effect is included. Concordia (6) notes in particular the inclusion of damper winding effect is most critically needed in the case of cross-compound generators. Byerly, Skooglund, and Keay (4) in an analog study reported improvement in stability obtained with a power damping control signal fed into the voltage regulator.

VanNess (31) used modern control theory, root loci approach, in his investigation of interconnected systems incorporating the turbine, generator, governor and tie-line controller. Laughton (21) took Heffron's modeling of the synchronous generator and its A-constants combining it into a state-space representation of a 30 mw generator and regulator. The dynamic stability was checked by noting the negativeness of the real part of the eigenvalues of the A-matrix from the state-space formulation of the differential equations.

Undrill (30) extends the work of Laughton analytically and in particular separating out the turbine damping and rotor damping effects. A four-machine system is analyzed

with methods given for extending to a higher number of machines.

Kekela and Firestone (17) present a study of under-excited operation, a problem associated with medium to long length transmission lines.

Schleif and White (28) give an analog study on oscillations prevalent between basically hydro and steam generating areas interconnected by long transmission lines.

III. SYSTEM STABILITY BY EIGENVALUES

Modern control theory (3) has permitted the investigation of stability of differential equations describing a system by means of root locus, Bode, or Nyquist plots, to name a few. Routh criterion approach determines stability by ascertaining the negativeness of the real part of the roots of the higher-order characteristic equation without factoring it.

A. Method of Analysis

The method employed to determine stability is to form the equations interrelating circuit values and motion as a series of first order differential equations. By means of Taylor's expansion, linearization around steady-state operating conditions is made and differential equations formed with the equations interrelated by variables defined as states of the vector space method (2). Placing these equations in matrix algebra notation (5) the required eigenvalues of such a reduced system matrix are the same as the roots of the characteristic equation. The negativeness of the real part of the eigenvalue assures stability; the more negative, the sooner the contribution of the associated term will die out. A complex conjugate root indicates oscillation with the higher frequencies associated with the larger imaginary parts.

B. Formulation of the Equation

In matrix algebra, the equations take the form $\dot{\underline{x}} = \underline{A} \underline{x} + \underline{D} \underline{u}$ where \underline{A} and \underline{D} are the matrices of constant coefficients, \underline{u} is the forcing function input vector and \underline{x} is the vector representing the state of the system variables. In this investigation the stability of the equations was determined by digital methods on an IBM 360/65 using Fortran admitting complex algebra programming. The real part of the eigenvalues of the A-matrix at various states must remain negative to insure that the solution equation decays to a stable final value as would be obtained in a real-time solution for $x(t)$. Thus the linearized equations thus formed give a good approximation to the resulting time solutions. It is this piecewise or indirect method of determining stability that is investigated further.

General information concerning the boundaries of stable regions is not gained from this method but the stability limits under a multitude of operating conditions can be accurately determined.

C. Eigenvalue Routine

The correctness of the investigation relies heavily on the eigenvalue solution routine for a non-symmetric matrix by the QR transform by Imad and VanNess (15). This is available as a Share program library routine. The main program

calls this library routine and the eigenvalues only were printed out in real and imaginary parts. Lacking a negative sign, the positive real part of any eigenvalue marks that particular power output and power factor setting, unstable.

The Share program was checked against the Krylov Method for non-symmetric matrices in Faddeeva* and checked to five decimal places. It is known, however, that the QR transform routine is dependent upon the A-matrix itself. The routine was dimensioned at 30 by 30.

D. Extent of the Investigation

A large modern single-shaft steam turbine driven (600 to 700 mw) generator was investigated. It was connected to an infinite bus through a typical long transmission line or tie line. It had an excitation control system and a prime-mover and governor system. The investigation follows Laughton (21) in per-unit on the machine base but varies in five ways:

1. The infinite bus is held at 1.0 pu on the machine base voltage and the effect of regulating by adjusting the generating terminal voltage is noted. Laughton held the terminal voltage at 1.0 pu.
2. The generator is large, typical of present day large units, whereas Laughton's generator was 30 mw.

*Faddeva (12, p. 156).

3. Laughton investigated stability with the voltage regulator and machine only. This investigation includes the prime mover and governor.
4. Effect of varying machine damping, D , is included.
5. Effect of varying the tie-line reactance, X -line, is included. A Pi-line equivalent network (see Figure 2) is used.

The method of extending the investigation to include a tie-line controller is given; also, Laughton's method of extending the study to a multi-machine system is included.

IV. SYNCHRONOUS GENERATOR

The phasor diagram covering the Parks direct and quadrature-axis representation of the synchronous generator (25, 26) is given in Figure 1. It is convenient to include the tie-line connecting the generator to the infinite bus in the same analysis. The equivalent adjusted Pi-line representation* per Figure 2 has been made by separate calculation.

The list of symbols and Fortran equivalent names are given in the Appendix A. The actual per-unit, (pu) values on the machine base used are found in Programs 1 and 2 of the Appendix C. It should be noted that since power factor is the independent variable and with generator power output held constant, pu current equals pu power output divided by the product of power factor times the pu terminal voltage. Rated three-phase mva and rated line-to-line terminal voltage are the two machine bases, both taken as 1.0 pu. Rated full-load power in pu is then 0.9 pu for a 0.90 power factor machine and must be so read into the program. The infinite bus voltage, V_b , is held at 1.0 pu as long as the terminal voltage, V_t , does not fall below 0.8 pu to maintain this condition. V -terminal must be determined by iterating with each change in power factor.

*Westinghouse Electric Corporation (33, p. 44).

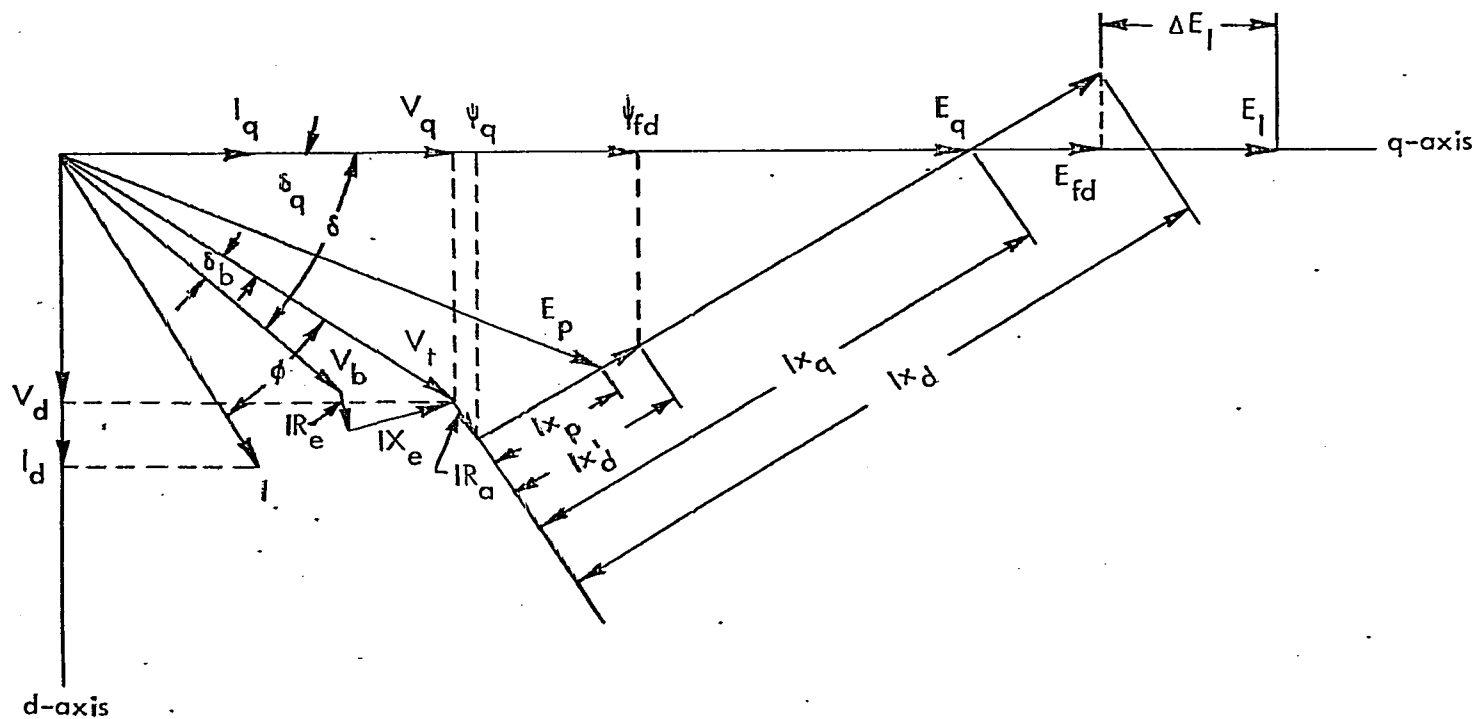


Figure 1. Phasor diagram of generator and tie-line

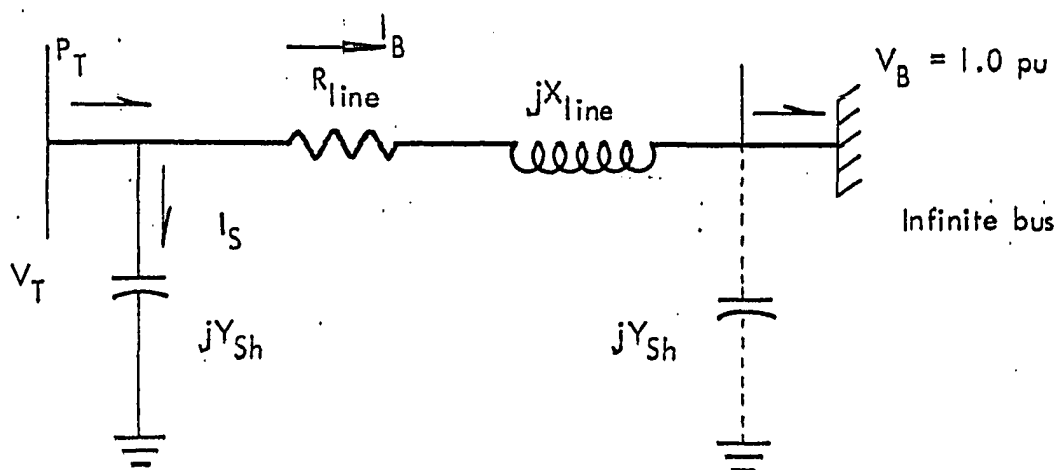


Figure 2. Model of transmission line for the calculation of the terminal voltage of the generator

It should be noted that the tie-line or transmission line inductive reactance often identified by x -line rather than x_e refers to its equivalent pu reactance on the machine base.

The effect of change of speed on generated voltages due to the damper winding has been assumed as negligible in this analysis. Balanced loads are assumed.

A. Operating Matrix Equations

When a Taylor-series expansion is formed about any operating point, for each of the equations (54-589), the resulting linear perturbation relationships of the first order may be expressed as in Equation 1. This matrix equation summarizes the relationship between all machine and system variables where the inputs are ΔT_m through the prime mover and Δv_{fd} through the excitation system. The controlled machine variables $\Delta \delta$, ΔV_t and ΔI , and all time-derivative quantities are in the first two equations. (See following page for Equation 1.)

B. Reduced System A-constants

Equation 1 has the matrix form.

$$\underline{F} = \underline{M} \underline{G} \quad (2)$$

or, in the expanded form:

[illegible]

(1)

$$\begin{bmatrix} \Delta T_m \\ \vdots \\ \vdots \\ 0 \\ \hline 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} & M_1 & & M_2 \\ & & & \\ \hline & & & \\ M_3 & & & M_4 \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \vdots \\ \vdots \\ \vdots \\ \hline \Delta I \\ \Delta v_d \\ \vdots \\ \vdots \\ \vdots \\ \Delta \psi_q \end{bmatrix} \quad (3)$$

and therefore the variables not of interest, Δv_d to $\Delta \psi_q$, may be eliminated by matrix reduction, yielding

$$\begin{bmatrix} \Delta T_m \\ \Delta v_{fd} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} & \\ M_1 - M_2 M_4^{-1} M_3 & \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \Delta \psi_{fd} \\ \Delta v_t \\ \Delta I \end{bmatrix} \quad (4)$$

Writing, for purposes of brevity, $R_d = r_d + r_e$, $X_d = x_d + x_e$ and continuing for R_q , X_q , X'_d , Equation 4 can be expanded as

$$\begin{bmatrix} \Delta T_m \\ \Delta v_{fd} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} M_p^2 + D_p - A_1 & -A_2 & 0 & 0 \\ -A_3 & -A_4(1 + \tau'_{dz} p) & 0 & 0 \\ -A_5 & -A_6 & 1 & 0 \\ -A_7 & -A_8 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \Delta \psi_{fd} \\ \Delta v_t \\ \Delta I \end{bmatrix} \quad (5)$$

where the relationships summarized by the constants $A_1 - A_8$, are given in Appendix B. These are the A-constants found in Laughton (21) which also are directly related to constants formed by Heffron and Phillips (14). Laughton found that eliminating resistances introduced a five percent error. With a fast computer the added correctness warranted retaining all resistances.

The matrix reduction was done algebraically to confirm Laughton's A-constants. It can be done by digital matrix reduction as well, switching rows so that the principal diagonal contains no zeros.

C. Feedback Control Scheme

The operating matrix Equation 1 and the reduced system matrix Equation 5 represent an open-loop system of one machine connected to an equivalent infinite bus system with two arbitrary forcing functions, the excitation system and the prime mover. Addition of further regulating equipment may be made with the field control responding to changes in terminal voltage, current and rotor angle. Additional equations, including the governor are

$$\Delta v_{fd} = -G_1(p) \{ \Delta v_t + G_2(p) \Delta I + G_3(p) \Delta \delta \} \quad (6)$$

and

$$\Delta T_m = -G_4(p) \Delta \delta \quad (7)$$

where $G_1(p), \dots, G_4(p)$ are the transfer functions of these respective sub-systems.

The block diagram of the above control scheme and machine with feed-back loops is given in Figure 3.

D. State-space Equations

It is now possible to rearrange the Equation 5 by state-space methods (21) to the required set of first-order equations. For linear constant-coefficient systems, the equations take the form

$$\dot{\underline{x}} = \underline{A} \underline{x} + \underline{D} \underline{u} \quad (8)$$

A first-order vector differential equation, where \underline{A} and \underline{D} are matrixes of constant coefficients, \underline{u} is the forcing-function inputs and \underline{x} the vector representing the state of the system variables.

Letting

$$x_1 = \Delta\delta$$

$$x_2 = \dot{x}_1 = \dot{\Delta\delta}$$

$$x_3 = \Delta\psi_{fd}$$

and control input variables,

$$u_1 = \Delta v_{fd}$$

$$u_2 = \Delta T_m ,$$

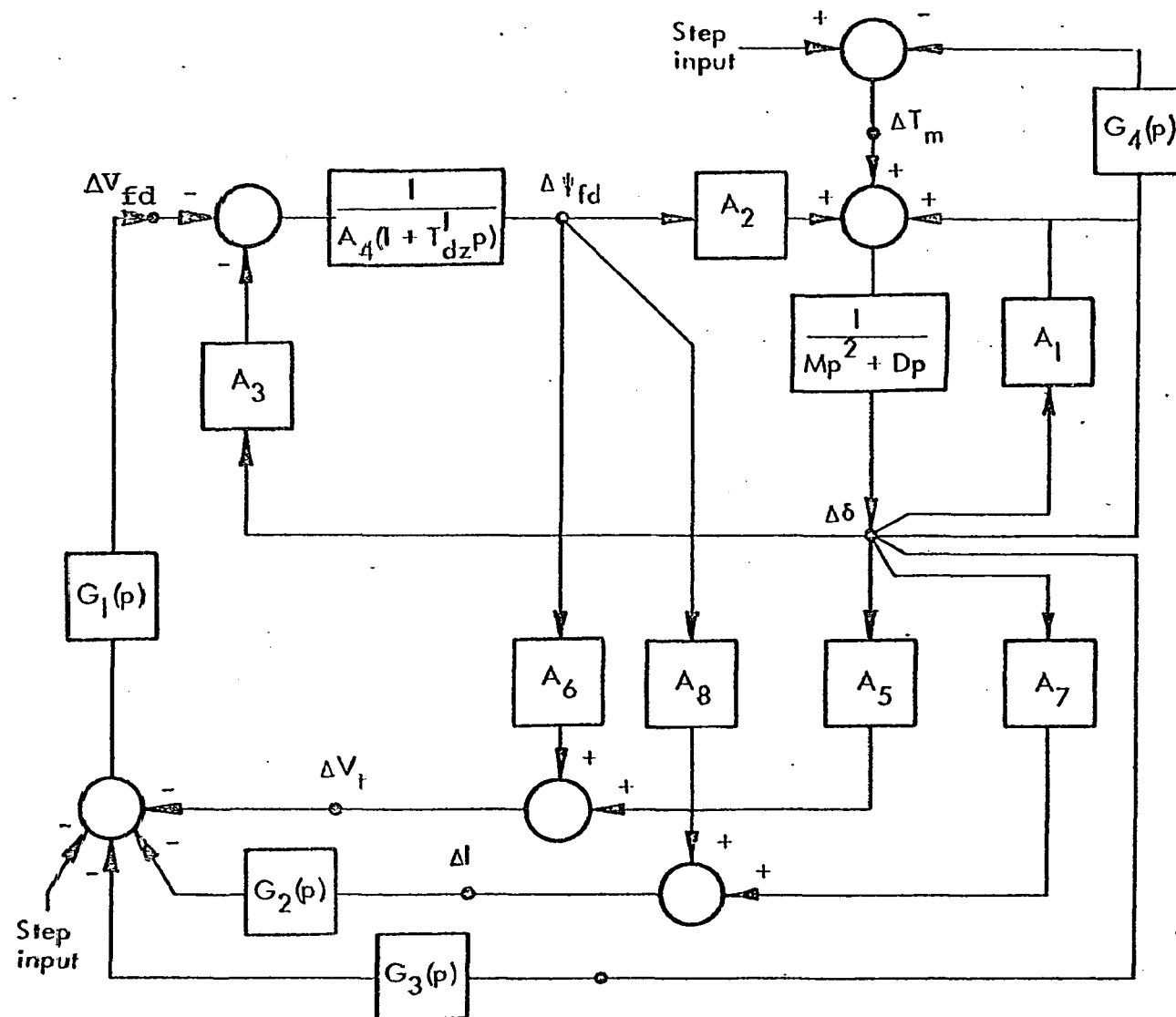


Figure 3. Block diagram of synchronous generator connected to an equivalent power system with multi-loop machine feedback control system

Equation 5 becomes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{A_1}{M} & \frac{-D}{M} & \frac{A_2}{M} \\ \frac{-A_3}{A_4 \tau'_{dz}} & 0 & \frac{-1}{\tau'_{dz}} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M} \\ \frac{-1}{A_4 \tau'_{dz}} & 0 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (9)$$

The output may also be represented by

$$\underline{y} = \underline{B} \underline{x} \quad (10)$$

in which case Equation 5 can be expanded by substituting

$y_1 = \Delta V_t$, $y_2 = \Delta I$ giving

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} A_5 & 0 & A_6 \\ A_7 & 0 & A_8 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (11)$$

The stability of the system by Equation 5 is assured when the forcing functions \underline{u} are finite values, only if all the eigenvalues of the coefficient matrix \underline{A} as found from

$$\text{Det}(\underline{A} - \lambda \underline{I}) = 0 \quad (12)$$

have negative real parts.

E. Damping

The system damping including that of the damper winding circuit is investigated at two values, $D = 2$ and $D = 3$. It does not include the effect of rotor damping currents on the generated voltage.

Concordia (6) states that except for the case of cross-compound generator sets, omitting the damper winding (or amortisseur winding) has little effect on steady-state stability. Likewise Laughton (21) states damping is of no great importance as to locating the stability limit.

Undrill (30) treats the damper winding separately with speed voltage effects. The necessity and accuracy of this addition is an area for separate study.

F. Saturation

Generator saturation manifests its effect in two ways, in the transient reactance and in the excitation requirements.

1. Reactances

Kilgore (18) has published the effect of saturation in depth. Based on Kilgore's data the transient reactance used for x'_d was taken at 0.93 times the saturated transient reactance at 1.0 pu current. All other values of reactances and open-circuit field time constants were based on unsaturated conditions.

2. Excitation requirements

The generator saturation effect is taken from Young and Webler (34). The equation for the increase in excitation voltage, ΔE_I per Figure 1 is given by

$$\Delta E_I = A_G (\epsilon^{B_G(E_p - 0.8)}) \quad (13)$$

where E_p is the voltage behind Potier reactance x_p and A_G and B_G are constants derived from taking two points on the saturation curve, Figure 4.

A_G and B_G , calculated separately and found to be 0.035 and 7.358 respectively, appear in this form in Program 1 of Appendix C. E_p is calculated as $E_p = V_T + Ix_p$ in complex form. Finally ΔE_I is added to E_{fd} giving the saturated or actual pu field voltage needed.

Generator field heating is a possible limitation on over-excited operation with the infinite bus held at 1.0 pu voltage.

G. Tie-line Reactance

The tie-line or transmission line is an equivalent Pi-representation given by Figure 2. The bus voltage is held at 1.0 pu of generator terminal voltage as long as the generator terminal voltage does not fall below 0.8 pu. By iteration, in Program 1, Appendix C, the terminal voltage is adjusted to give $V_b = 1.0$ pu, without tap-change, before

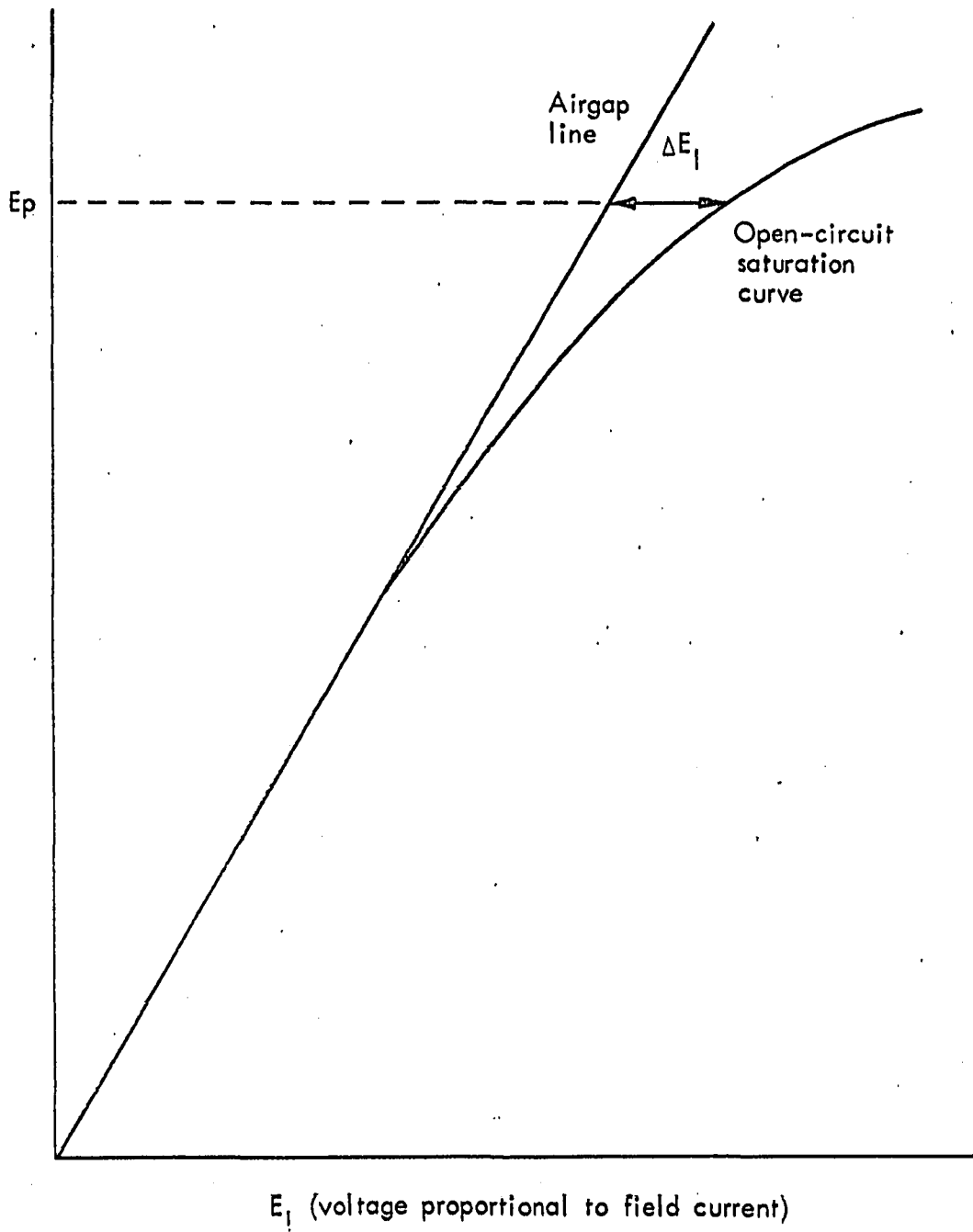


Figure 4. Representation of saturation of generator

further calculations are made.

Two values of tie-line impedance were investigated. Since the value of the tie-line reactance has a great effect on stability, tabulations are all identified by the tie-line reactance used, namely, $X\text{-LINE} = 0.414$ pu or $X\text{-LINE} = 0.718$ pu. These are Pi-line adjusted values. Programs 1 and 2 of Appendix C input values that are associated with the lower and higher tie-line reactances, respectively. The use of shunt capacitors could easily be added as a separate study.

V. EXCITATION SYSTEM

The Institute of Electrical and Electronics Engineers power generating excitation systems subcommittee has formulated and reported recommended computer representations for four major types of excitation systems (16). The constants and symbols used with block diagrams, per unit notation, and exciter saturation representation used below follow this committee report.

A. Type 1 System

Figure 5 is the block diagram of the excitation system. One of the signals initially summed is the time derivative of electric power. The state-space equations for the type 1 system will be developed and then the power signal treated separately.

Following Anderson and Fouad (1) the type 1 system may be redrawn as Figure 6.

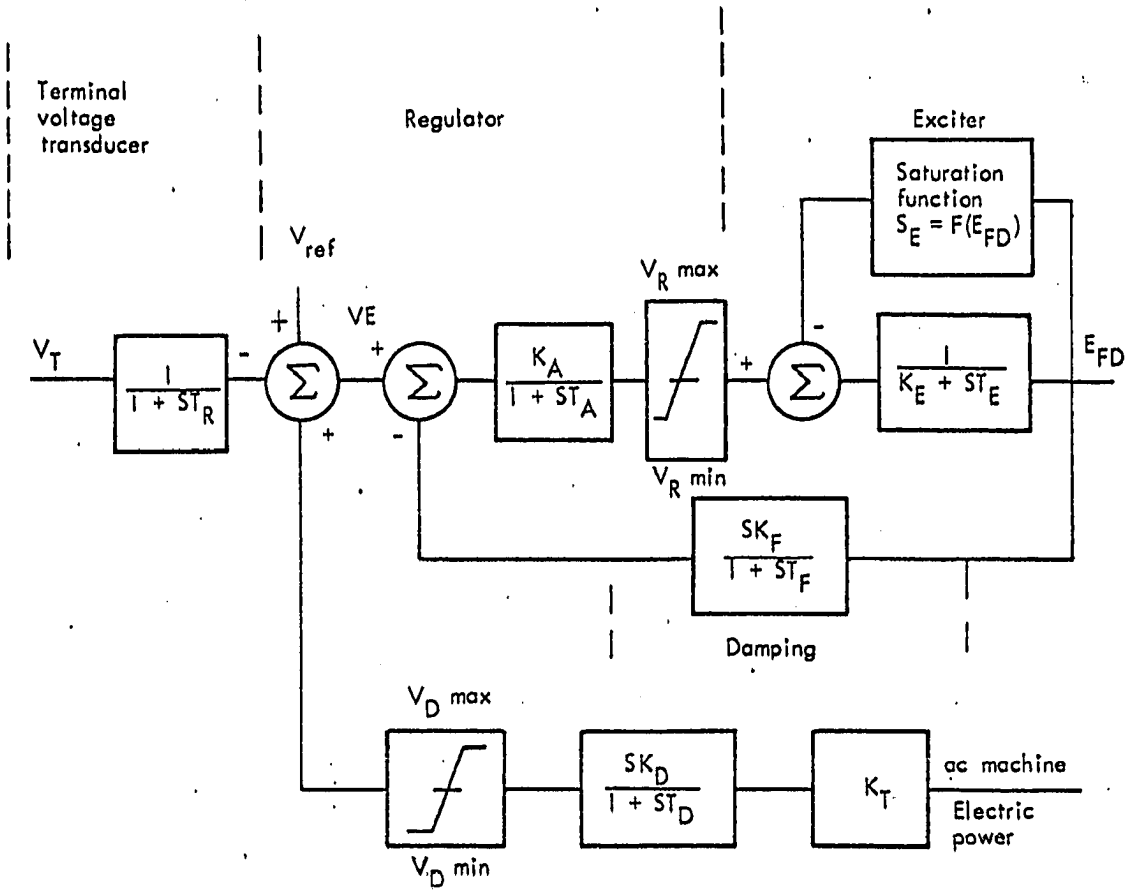
Now

$$\frac{V_F}{V_E} = \frac{G_2 G_3}{1 + G_3 H_1 + G_2 G_3 H_2} \quad (14)$$

where

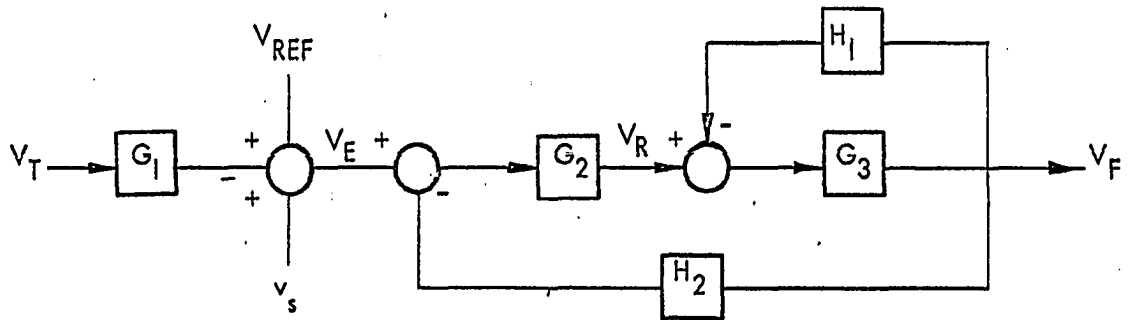
$$V_E = V_{REF} + V_S - G_1 V_T$$

Substituting and rearranging



- E_{FD} Exciter output voltage (applied to generator field)
 K_A Regulator gain
 K_E Exciter constant related to self-excited field
 K_F Regulator stabilizing circuit gain
 T_A Regulator amplifier time constant
 T_E Exciter time constant
 T_F Regulator stabilizing circuit time constant
 T_R Regulator input filter time constant
 V_R Regulator output voltage
 V_T Generator terminal voltage
 K_D Damping gain
 T_D Damping time constant
 VE Intermediate variable point

Figure 5. Type 1 excitation system block diagram



Where $G_1 = \frac{1}{1 + ST_R}$

$$G_2 = \frac{K_A}{1 + ST_A}$$

$$G_3 = \frac{1}{K_E + ST_E}$$

$$H_1 = S_E$$

$$H_2 = \frac{SK_F}{1 + ST_F}$$

Figure 6. Simplified block diagram of type 1 excitation system

$$\frac{V_F}{V_E} = \frac{K_A(1+ST_F)}{(1+ST_A)(K_E+ST_E)(1+ST_F) + S_E(1+ST_A)(1+ST_F) + K_A K_F S} \quad (15)$$

The right-hand side denominator, call it $\phi(s)$, is the characteristic equation. Rearranging

$$\phi(s)V_F = K_A(1+ST_F)V_E \quad (16)$$

where

$$\begin{aligned} \phi(s) = & T_A T_E T_F s^3 + [K_E T_A T_F + T_E(T_A + T_F) + S_E T_A T_F] s^2 \\ & + [(K_E + S_E)(T_A + T_F) + T_E + K_A K_F] s + (K_E + S_E) \end{aligned} \quad (17)$$

dividing by $(T_A T_E T_F)$ and taking the inverse Laplace transform gives

$$\overset{...}{v}_f + a_1 \overset{..}{v}_f + a_2 \overset{\cdot}{v}_f + a_3 v_f = b_0 \overset{...}{v}_e + b_1 \overset{..}{v}_e + b_2 \overset{\cdot}{v}_e + b_3 v_e$$

where

$$\begin{aligned} a_1 &= \frac{T_E(T_A + T_F) + T_A T_F(K_E + S_E)}{T_A T_E T_F} \\ a_2 &= \frac{(K_E + S_E)(T_A + T_F) + T_E + K_A K_F}{T_A T_E T_F} \\ a_3 &= \frac{K_E + S_E}{T_A T_E T_F} \end{aligned} \quad (18)$$

$$b_0 = b_1 = 0$$

$$b_2 = \frac{K_A}{T_A T_E}$$

$$b_3 = \frac{K_A}{T_A T_E T_F}$$

Rewriting Equation 18 terms of p-operator as

$$(p^3 + a_1 p^2 + a_2 p + a_3) v_f =$$

$$(b_0 p^3 + b_1 p^2 + b_2 p + b_3) v_e$$

or

$$\begin{aligned} p^3(v_f - b_0 v_e) + p^2(a_1 v_f - b_1 v_e) + p(a_2 v_f - b_2 v_e) \\ + (a_3 v_f - b_3 v_e) = 0 \end{aligned} \quad (19)$$

and rearranging after dividing by p^3 we get

$$\begin{aligned} v_f - b_0 v_e = \frac{1}{p}(b_1 v_e - a_1 v_f) + \frac{1}{p} \left[\frac{1}{p}(b_2 v_e - a_2 v_f) + \frac{1}{p} \left(\frac{1}{p}(b_3 v_e - a_3 v_f) \right) \right] \end{aligned} \quad (20)$$

calling $x_4 = v_f - b_0 v_e$, following the underscoring brackets

$$\dot{x}_5 = \dot{x}_4 + (a_1 v_f - b_1 v_e)$$

$$\dot{x}_6 = \dot{x}_5 + (a_2 v_f - b_2 v_e) \quad (21)$$

To place the equations in state-space form $\dot{\underline{x}} = \underline{A} \underline{x} + \underline{D} \underline{u}$,
noting that $v_f = x_4 + b_o v_e$ now

$$x_6 \equiv \frac{1}{p}(b_3 v_e - a_3 v_f)$$

$$p x_6 = b_3 v_e - a_3 v_f$$

(22)

$$\dot{x}_6 = b_3 v_e - a_3 (x_4 + b_o v_e)$$

$$\dot{x}_6 = -a_3 x_4 + (b_3 - a_3 b_o) v_e$$

and

$$x_5 \equiv \frac{1}{p}(b_2 v_e - a_2 v_f + x_6)$$

$$\dot{x}_5 = b_2 v_e - a_2 (x_4 + b_o v_e) + x_6 \quad (23)$$

$$\dot{x}_5 = -a_2 x_4 + x_6 + (b_2 - a_2 b_o) v_e$$

and

$$x_4 \equiv \frac{1}{p}(b_1 v_e - a_1 v_f) + x_5$$

$$\dot{x}_4 = b_1 v_e - a_1 (x_4 + b_o v_e) + x_5 \quad (24)$$

$$\dot{x}_4 = -a_1 x_4 + x_5 + (b_1 - a_1 b_o) v_e$$

In matrix form $\dot{\underline{x}} = \underline{A} \underline{x} + \underline{D} \underline{u}$, noting $b_o = b_1 = 0$

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 \\ -a_2 & 0 & 1 \\ -a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} b_1 - a_1 b_0 \\ b_2 - a_2 b_0 \\ b_3 - a_3 b_0 \end{bmatrix} \cdot \begin{bmatrix} v_e \end{bmatrix} \quad (25)$$

Addition of the three "inputs"

$$V_E = \frac{-V_T}{1 + ST_R} + V_{REF} + V_S$$

from which

$$\dot{v}_e + \frac{v_e}{T_R} = -V_T + \frac{v_{ref} + v_s}{T_R}$$

calling $x_7 = v_e$ this would change the matrix to

$$\begin{bmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 & (b_1 - a_1 b_0) \\ -a_2 & 0 & 1 & (b_2 - a_2 b_0) \\ -a_3 & 0 & 0 & (b_3 - a_3 b_0) \\ 0 & 0 & 0 & \frac{-1}{T_R} \end{bmatrix} \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-1}{T_R} \end{bmatrix} \cdot \begin{bmatrix} v \end{bmatrix} \quad (26)$$

where $v = -v_t + v_{ref} + v_s$.

B. Power Damping Signal

The lower blocks in Figure 5 inputs a damping power signal. The time constant T_R is zero. This alters as follows:

$$V_E = -V_T + V_{REF} + \frac{K_T S K_D P}{1 + S T_D}$$

from which

$$v_e + T_D \dot{v}_e = -v_t + v_{ref} + K_T K_D \frac{dP}{d\delta} \cdot \frac{d\delta}{dt}$$

and

$$\dot{v}_e = \frac{-v_e - v_t + v_{ref}}{T_D} + \frac{K_T K_D}{T_D} \frac{dP}{d\delta} \cdot \frac{d\delta}{dt} \quad (27)$$

Now

$$\frac{d\delta}{dt} = x_2 = \dot{x}_1 = \Delta \dot{\delta}$$

from the machine state-space equation and $\frac{dP}{d\delta}$ can be calculated from Concordia*

$$P = \frac{EV_T}{x'_d} \sin \delta + \left(\frac{1}{x'_q} - \frac{1}{x'_d} \right) \frac{V_T^2}{2} \sin 2\delta$$

and

$$\frac{dP}{d\delta} = \frac{EV_T \cos \delta}{x'_d} + \left(\frac{1}{x'_q} - \frac{1}{x'_d} \right) V_T^2 \cos 2\delta \quad (28)$$

* Concordia (8, p. 38).

where

$$E = \psi_d + x'_d i_d$$

The regulator can now be combined with the machine. K_A inputs the signals to excitation system level similar to Laughton's* G_1 function. In this machine a generator field voltage of 274 volts produces 1.0 per unit generator volts on the air-gap line. This is the K_2 scaling factor, per Laughton.*

C. Saturation

Figure 7 shows the meaning of the saturation function and how it is actually calculated. In the same manner as Equation 13 was used to calculate ΔE_I for the generator saturation, in a like manner S_E can be calculated from two values from the exciter saturation curve or equivalent data.

$$S_E = A_E (\epsilon^{B_E (E_{FDS}^{-.8})}) \quad (29)$$

In this exciter A_E was calculated to be 0.235 and B_E was calculated to be 0.214. S_E was limited to give a maximum of 0.835 in the program for cases of overexcited operation.

*Laughton (21, p. 335).

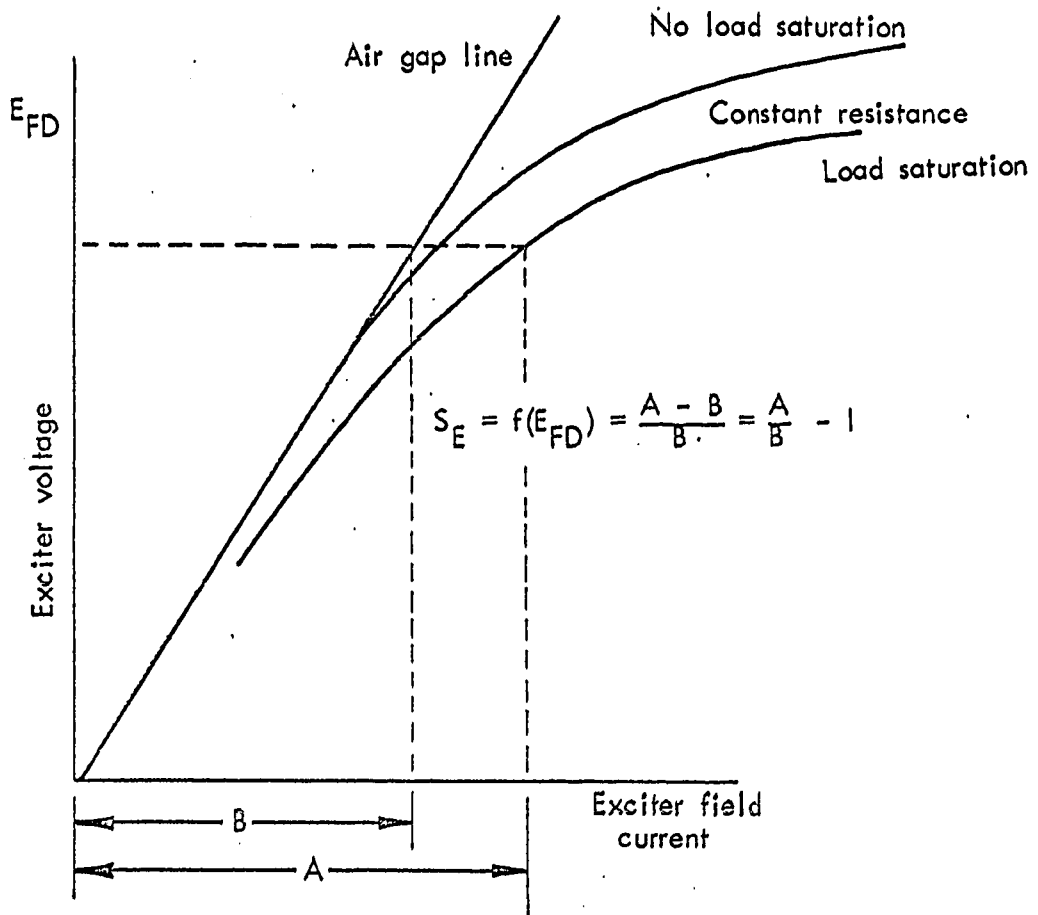


Figure 7. Calculation of exciter saturation

D. State-space Equations

Equation 30 gives the resulting state-space matrix formulation. Note $b_0 = b_1 = 0$.

\underline{A} - matrix

$$\begin{bmatrix} \dot{x} \end{bmatrix} = \begin{bmatrix} & 1 & & & & & \\ \frac{A_1}{M} & \frac{-D}{M} & \frac{A_2}{M} & & & & \\ \frac{-A_3}{A_4 T' dz} & & \frac{-1}{T' dz} & \frac{-K_2}{A_4 T' dz} & & & \\ & & & -a_1 & 1 & & \\ & & & -a_2 & & 1 & b_2 \\ & & & -a_3 & & & b_3 \\ & \frac{K_T K_D}{T_D} \frac{dP}{d\delta} & & & & & \frac{-1}{T_D} \end{bmatrix} \begin{bmatrix} x \\ x_1 = \Delta \delta \\ x_2 = \Delta \dot{\delta} \\ x_3 = \Delta \psi_{fd} \\ x_4 = \Delta V_f \\ x_5 \\ x_6 \\ x_7 = \Delta V_e \end{bmatrix}$$

$$+ \begin{bmatrix} D \\ \frac{1}{M} \\ \\ \\ \\ \\ \frac{-1}{T_D} \end{bmatrix} \cdot \underline{U} \begin{bmatrix} \Delta \dot{T}_M \\ \Delta V \end{bmatrix} \quad (30)$$

Where $V = -\Delta V_f + \Delta V_{ref}$

x_4, x_5, x_6, x_7 derive from state formulation of type I excitation system.

VI. GOVERNOR SYSTEM

The single-shaft steam turbine driving the generator is given by block diagram representation in Figure 8 which includes the details of the prime mover and governor. State-space equations are formulated. The addition of an auxiliary control signal, tie-line control, is treated separately. Gain K_1 has been divided by rated mw to place it in per-unit level.

A. Single-shaft Governor Representation

State-space variable approach from Anderson and Fouad (1) will be followed. The steam system dynamics includes steam and reheat time constants and the fraction of reheat. The governor includes water hammer correction, the servo valve motor, and control system time constants. The block diagram of the single-shaft governor is given in Figure 8.

B. State-space Equations

1. Steam-system dynamics

The transfer function of the steam system Figure 9 by parallel programming is:

$$G(s) = \frac{1 + K_2 T_5 s}{(1 + T_4 s)(1 + T_5 s)} \quad (31)$$

By partial fraction expansion

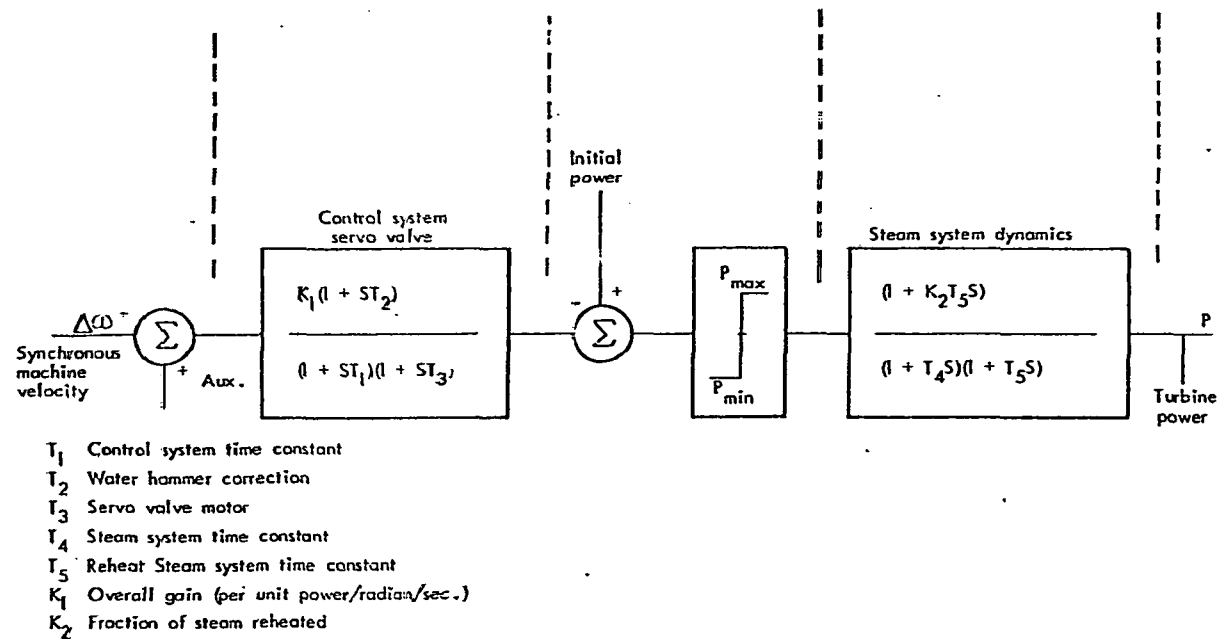


Figure 8. Block diagram of governor system

$$G(s) = \frac{A}{1 + T_4 s} + \frac{B}{1 + T_5 s} \quad (32)$$

Solving for A and B

$$A = \frac{T_4 - K_2 T_5}{T_4 - T_5}$$

$$B = \frac{T_5 (1 - K_2)}{T_5 - T_4}$$

and substituting in Equation 32 gives

$$G(s) = \frac{(T_4 - K_2 T_5)/T_4 (T_4 - T_5)}{s + 1/T_4} + \frac{(1 - K_2)/(T_5 - T_4)}{s + 1/T_5} \quad (33)$$

calling

$$A' = (T_4 - K_2 T_5)/T_4 (T_4 - T_5)$$

$$B' = (1 - K_2)/(T_5 - T_4)$$

the branches take the forms

$$\frac{u}{x} = \frac{A'}{s + 1/T_4}$$

and

$$\frac{u}{x} = \frac{B'}{s + 1/T_5}$$

Combining as in Figure 10 the state-space variables are:

$$\dot{x}_8 = \frac{-1}{T_4} x_8 + u$$

$$\dot{x}_9 = \frac{-1}{T_5} x_9 + u$$

and

$$\dot{x} = A'x_8 + B'x_9 \quad (34)$$

2. Governor control system and servo valve

The transfer function of the governor itself, Figure 8 by parallel programming is:

$$G(s) = \frac{K_1(1 + T_2s)}{(1 + T_1s)(1 + T_3s)} \quad (35)$$

By partial fraction expansion

$$G(s) = \frac{B_1}{1 + T_1s} + \frac{B_2}{1 + T_3s} \quad (36)$$

Solving for B_1 and B_2

$$B_1 = \frac{K_1(T_1 - T_2)}{(T_1 - T_3)}$$

$$B_2 = \frac{K_1(T_3 - T_2)}{(T_3 - T_1)}$$

Substituting in Equation 36 gives

$$G(s) = \frac{K_1(T_1 - T_2)/T_1(T_1 - T_3)}{s + 1/T_1} + \frac{K_1(T_3 - T_2)/T_3(T_3 - T_1)}{s + 1/T_3} \quad (37)$$

calling

$$B'_1 = K_1(T_1 - T_2)/T_1(T_1 - T_3)$$

$$B'_2 = K_1(T_3 - T_2)/T_3(T_3 - T_1)$$

Combining as per Figure 11 the state-space variables are

$$\dot{x}_{10} = -\frac{1}{T} x_{10} + u_{in}$$

$$\dot{x}_{11} = -\frac{1}{T_3} x_{11} + u_{in}$$

$$u' = B'_1 x_{10} + B'_2 x_{11} \quad (38)$$

Combining the two transfer functions per Figure 12

$$u_{in} = -\Delta\dot{\delta} + \omega_s = \omega_s - x_2 \quad (39)$$

where $x_2 = \Delta\dot{\delta}$; ω_s = auxiliary signal. Substituting in Equation

$$\dot{x}_{10} = -\frac{1}{T_1} x_{10} - x_2 + \omega_s$$

$$\dot{x}_{11} = \frac{1}{T_3} x_{11} - x_2 + \omega_s$$

Solving for \dot{x}_8 and \dot{x}_9

$$\begin{aligned} u_{\Delta} &= P_{\Delta 0} - u' \\ &= P_{\Delta 0} - B'_1 x_{10} - B'_2 x_{11} \end{aligned} \quad (40)$$

where subscript Δ refers to a small increment of that variable

$$\dot{x}_8 = -\frac{1}{T_4} x_8 + u_{\Delta}$$

$$= -\frac{1}{T_4} x_8 - B'_1 x_{10} - B'_2 x_{11} + P_{\Delta 0}$$

and

$$\dot{x}_9 = -\frac{1}{T_5} x_9 - B'_1 x_{10} - B'_2 x_{11} + P_{\Delta 0} \quad (41)$$

State-space equations for governor system are

$$\dot{x}_8 = -\frac{1}{T_4} x_8 - B'_1 x_{10} - B'_2 x_{11} + P_{\Delta 0}$$

$$\dot{x}_9 = -\frac{1}{T_5} x_9 - B'_1 x_{10} - B'_2 x_{11} + P_{\Delta 0}$$

(42)

$$\dot{x}_{10} = -\frac{1}{T_1} x_{10} - x_2 + \omega_s$$

$$\dot{x}_{11} = -\frac{1}{T_3} x_{11} - x_2 + \omega_s$$

The final combined state-space formulation of generator, excitation system and governor control system is given in Equation 43.

\dot{x}_1
\dot{x}_2
\dot{x}_3
\dot{x}_4
\dot{x}_5
\dot{x}_6
\dot{x}_7
\dot{x}_8
\dot{x}_9
\dot{x}_{10}
\dot{x}_{11}

1	2	3	4	5	6	7	8	9	10	11
1	1									
2	$\frac{A_1}{M}$	$\frac{-D}{M}$	$\frac{A_2}{M}$							
3	$\frac{-A_3}{A_4 T dz}$	$\frac{-1}{T dz}$	$\frac{-K_2}{A_4 T dz}$							
4			$-a_1$	1						
5			$-a_2$		1	b_2				
6			$-a_3$			b_3				
7	$\frac{-A_5}{T_D}$	$\frac{-A_6}{T_D}$				$\frac{1}{T_D}$				
8							$-\frac{1}{T_4}$			
9							$-\frac{1}{T_5}$	$-B'_1$	$-B'_2$	
10		-1						$-\frac{1}{T_1}$	$-B'_1$	$-B'_2$
11		-1							$-\frac{1}{T_3}$	

x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
x_9
x_{10}
x_{11}

$$K_T K_D \frac{dp}{dT} \frac{d\delta}{dT_D}$$

1	
2	$\frac{1}{M}$
3	
4	
5	
6	
7	
8	
9	
10	
11	

$$\Delta T_M$$

1	
2	
3	
4	
5	
6	
7	
8	1
9	1
10	
11	

$$\Delta P_0$$

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	1
11	1

$$\frac{\Delta \omega_5}{\Delta V_5}$$

(4.3)

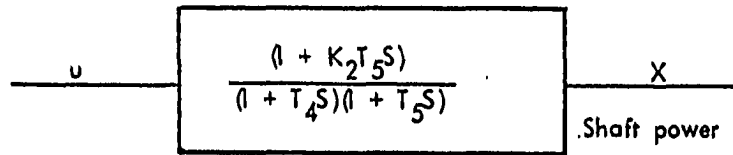


Figure 9. Transfer function of steam system

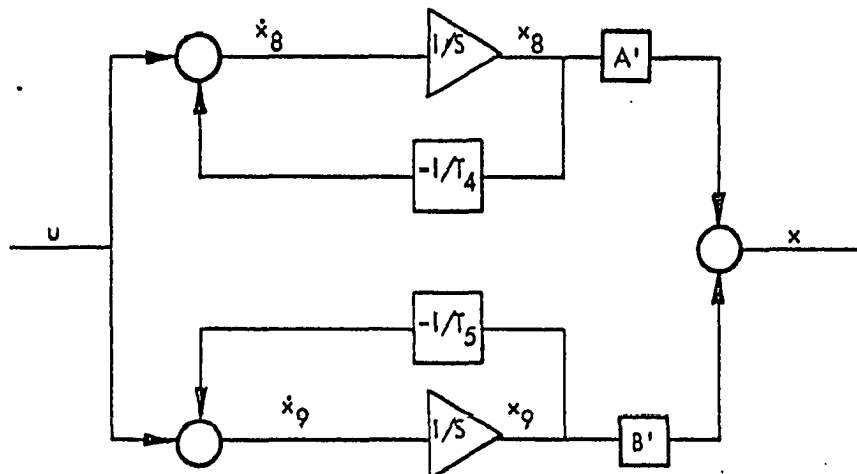


Figure 10. State-space diagram of steam system

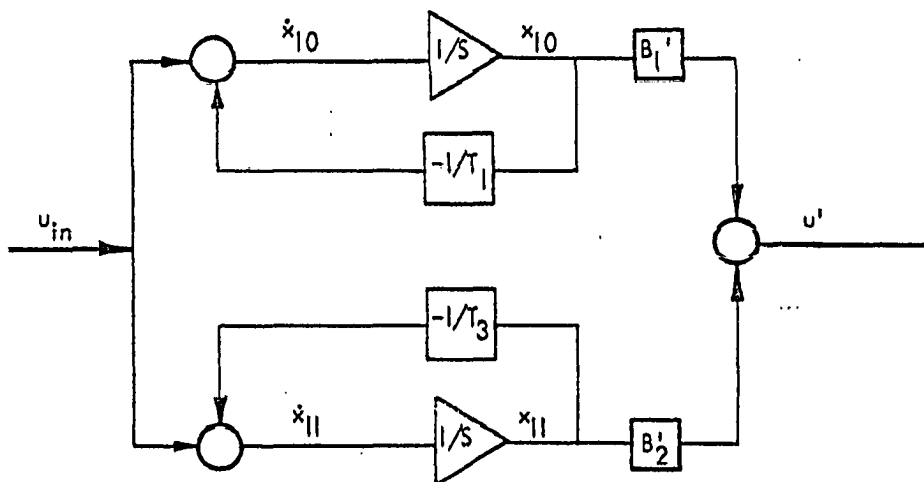


Figure 11. State-space diagram of control system and servo valve

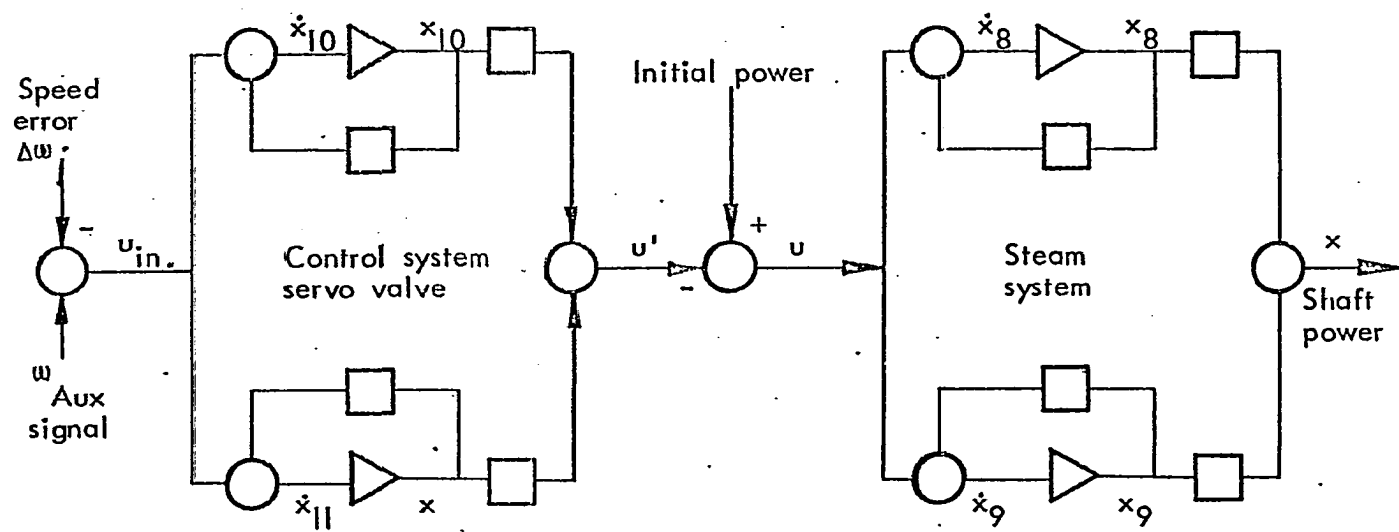


Figure 12. State space diagram of governor system

VII. TIE-LINE CONTROLLER

For area control, a tie-line controller can be added. The following method by VanNess gives the desired additional equations.

A. Tie-line Controller Representation

Tie-line, load frequency, control signals can be fed into the governor input as auxiliary signals, see Figure 13. VanNess' (31) speed-droop characteristic $1/R$ is separated from K_1 gain in the previous discussion. P_C is the reset control signal.

B. State-space Equations

The additions to the previous matrix equations by the controller are given by

$$\dot{X}_R = K_R X_P \quad (44)$$

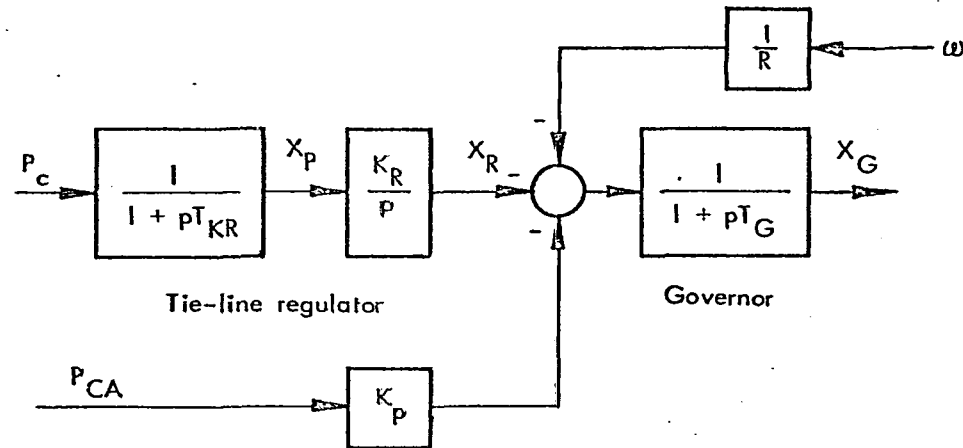
$$T_R \dot{X}_P + X_P = P_C$$

$$\dot{X}_P = \frac{P_C}{T_R} - \frac{X_P}{T_R} \quad (45)$$

$$T_G \dot{X}_G + X_G = -K_P P_{CA} - \frac{\omega}{R} - X_R ; \omega = x_2$$

$$\dot{X}_G = -\frac{X_G}{T_G} - \frac{X_R}{T_G} - \frac{x_2}{RT_G} - \frac{K_P P_{CA}}{T_G} \quad (46)$$

These additions to the state-space formulation are given by Equation 47.



R = Steady-state speed regulation
 K = Proportional gain of the regulator
 K_p = Reset gain of the regulator
 P_c = Reset control signal
 P_{CA} = Proportional control signal
 T_G = Governor time constant
 T_{KR} = Time constant associated with reset control signal
 X_G, X_p, X_R = Intermediate variables in block diagram

Figure 13. Block diagram of addition of tie-line controller

$$\begin{bmatrix} \dot{\delta} \\ \dot{X}_R \\ \dot{X}_p \\ \dot{X}_G \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & & K_R & \\ & & \frac{-1}{T_{KR}} & \\ \frac{-1}{RT_G} & \frac{-1}{T_G} & & \frac{-1}{T_G} \end{bmatrix} \cdot \begin{bmatrix} \omega \\ X_R \\ X_p \\ X_G \end{bmatrix} + \begin{bmatrix} & \\ & \\ & \frac{1}{T_{KR}} \\ \frac{-K_p}{T_G} & \end{bmatrix} \cdot \begin{bmatrix} P_{CA} \\ P_c \end{bmatrix} \quad (47)$$

VIII. MULTI-MACHINE PROBLEM

The analysis can be extended to cover two or more machines with the use of matrix algebra. Undrill (30) uses the terminal voltage of one machine as reference. Laughton (21) uses a separate infinite busbar reference axis. See Figure 14. The extension to the two machines is included for completeness following Laughton.

A. n-Machine Analysis

The equations which relate the machine variables and acceleration are

$$\begin{aligned} v_{qi} &= V_{ti} \cos \delta_{Ii} \\ v_{di} &= V_{ti} \sin \delta_{Ii} \end{aligned} \quad (48)$$

where $i = 1, 2, \dots, n$ (for 2 machines, $n = 2$).

Now the armature current of a generator is a function of all the other armature currents, it is related to its direct and quadrature reference frame and the system reference axis herein called V_b for simplicity.

Currents are defined $I_i = a_i + jb_i$ with

$$I_i^2 = a_i^2 + b_i^2 \quad (49)$$

where a_i and b_i are variables satisfying network laws.

Now matrix equations for node-voltage network approach

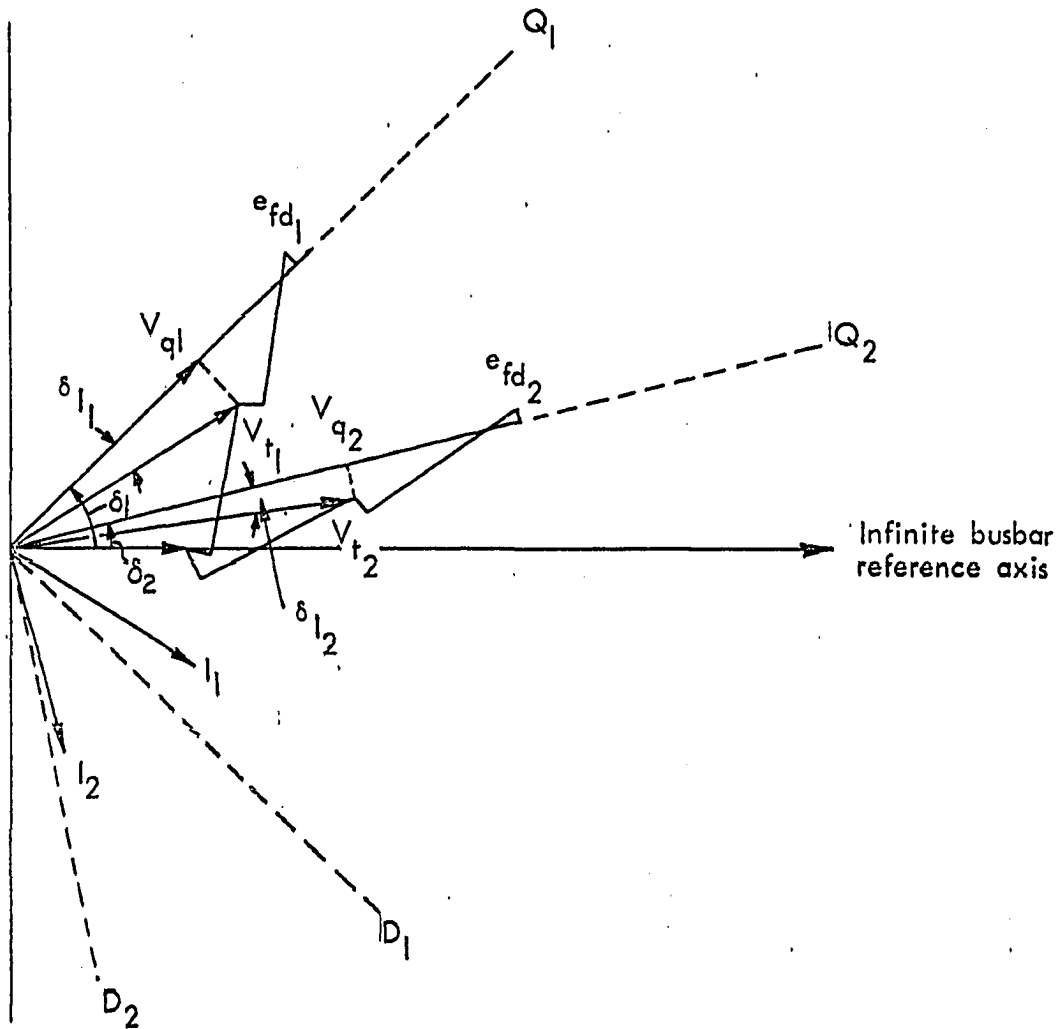


Figure 14. Phasor diagram of a two-machine system

are $\underline{I} = \underline{Y} \underline{V}$ where \underline{Y} = network node admittance matrix and \underline{V} the matrix of terminal voltages. Defining $\underline{Y} = \underline{G} - j\underline{B}$ and $\underline{V} = \underline{e} + j\underline{f}$;

$$a_i + jb_i = \sum_{m=1}^n (G_{im} - jB_{im})(e_m + jf_m),$$

where

$$e_m = V_{tm} \cos(\delta_m - \delta_{Im})$$

and

$$f_m = V_{tm} \sin(\delta_m - \delta_{Im})$$

therefore the final system equations are

$$\begin{aligned} a_i &= \sum_{m=1}^n \{G_{im} V_{tm} \cos(\delta_m - \delta_{Im}) + B_{im} V_{tm} \sin(\delta_m - \delta_{Im})\} \\ b_i &= \sum_{m=1}^n \{G_{im} V_{tm} \sin(\delta_m - \delta_{Im}) - B_{im} V_{tm} \cos(\delta_m - \delta_{Im})\} \end{aligned} \quad (50)$$

The operating point may be selected by means of an iterative matrix analysis (29) and perturbation relationships developed about this point as before.

For two machines on an infinite bus, the perturbation equations in matrix form are shown in Equation 51.

The constants k_{ij} in Equation 51 are the constants involved in the perturbation equations corresponding to Equation 50; thus

Δ^1_{m1}	$M_1 p^2 + D_1 p$	$- \dot{q}_1$	\dot{d}_1	$\dot{q}_1 - \dot{d}_1$
Δ^1_{m2}	$M_2 p^2 + D_2 p$	$(1 + \tau_{ff} p) - \tau_{ff} p (\dot{x}_{d1} - \dot{x}'_{d1})$	$-\frac{\dot{v}_{d1}}{\dot{v}_{l1}} - \frac{\dot{v}_{q1}}{\dot{v}_{l1}}$	$-\frac{\dot{d}_1}{\dot{l}_1}$
				$-\frac{\dot{q}_1}{\dot{l}_1}$
	$-\sin \theta_{l1}$		x_{d1}	r_{q1}
	$-\cos \theta_{l1}$		$x_{d1} - x'_{d1}$	$-x_{q1}$
			r_{d1}	
			x_{d1}	x_{q1}
				$-\frac{q_1}{l_1}$
	$-K_{22}$	$-K_{24}$	$-K_{21}$	$-K_{23}$
	$-K_{12}$	$-K_{14}$	$-K_{11}$	$-K_{13}$
			$-\sin \theta_{l2}$	
			$-\cos \theta_{l2}$	
	$-K_{42}$	$-K_{44}$	$-K_{41}$	$-K_{43}$
	$-K_{32}$	$-K_{34}$	$-K_{31}$	$-K_{33}$

$-i_{q2} \quad i_{d2} \quad i_{q2} \quad -i_{d2}$ $(1 + \tau_{f2}^p) \quad -\tau_{f2}^p(x_{d2} - x'_{d2})$ $-\frac{v_{d2}}{V_{f2}} \quad -\frac{v_{q2}}{V_{f2}}$				Δb_1 Δb_2 $\Delta \tau_{fd1}$ $\Delta \tau_{fd2}$ ΔV_{f1} ΔV_{f2} Δi_1 Δi_2
$-V_{f1} \cos \delta_{f1}$ $V_{f1} \sin \delta_{f1}$				Δv_{d1} Δv_{q1} $\Delta \tau_{fd1}$ Δi_{d1} Δi_{q1} $\Delta \tau_{d1}$ $\Delta \tau_{q1}$ ω_1 ωb_1 Δb_1
$-\frac{b_1}{T_1}$ 1	K_{22} K_{12}		K_{24} K_{14}	(51)
			$-V_{f2} \cos \delta_{f2}$ $V_{f2} \sin \delta_{f2}$	Δv_{d2} Δv_{q2} $\Delta \tau_{fd2}$ Δi_{d2} Δi_{q2} $\Delta \tau_{d2}$ $\Delta \tau_{q2}$ ω_2 ωb_2 Δb_2
		1 -1 -1 1 -1	x_{d2} $x_{d2} - x'_{d2}$ i_{d2} x_{d2} x_{q2} x_{q2}	i_{q2} $-x_{q2}$ 1 1 1 1
	K_{42} K_{32}		$-\frac{a_2}{T_2} \quad -\frac{b_2}{T_2}$ 1 1	K_{44} K_{34}

$$\Delta a_j = \sum_k \{ G_{jk} \cos(\delta_k - \delta_{Ik}) + B_{jk} \sin(\delta_k - \delta_{Ik}) \} \Delta V_{tk} \\ + V_{tk} \{ -G_{jk} \sin(\delta_k - \delta_{Ik}) + B_{jk} \cos(\delta_k - \delta_{Ik}) \} (\Delta \delta_k - \Delta \delta_{Ik})]$$

$$\Delta b_j = \sum_k \{ G_{jk} \sin(\delta_k - \delta_{Ik}) - B_{jk} \cos(\delta_k - \delta_{Ik}) \} \Delta V_{tk} \\ + V_{tk} \{ G_{jk} \cos(\delta_k - \delta_{Ik}) + B_{jk} \sin(\delta_k - \delta_{Ik}) \} (\Delta \delta_k - \Delta \delta_{Ik})]$$

$$j, k = 1, 2$$

or in matrix form

$$\begin{bmatrix} \Delta a_1 \\ \Delta b_1 \\ \Delta a_2 \\ \Delta b_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} \Delta V_{t1} \\ (\Delta \delta_1 - \Delta \delta_{I1}) \\ \Delta V_{t2} \\ (\Delta \delta_2 - \Delta \delta_{I2}) \end{bmatrix} \quad (52)$$

The coefficient matrix can be reduced by means of the Kron* reduction the same as for a single machine connected to an infinite bus. Writing the coefficient matrix in Equation 51 as

$$\underline{M} = \begin{bmatrix} M_1(p) + M_2 & M_3(p) + M_4 \\ M_5 & M_6 \end{bmatrix} \quad (53)$$

where $M_1(p)$ and $M_3(p)$ contain terms involving the differential

* Kron (20, p. 378).

operator p only. The reduced coefficient matrix representing the system of interest, represented by the first eight variables is

$$M_{\text{reduced}} = \{M_1(p) - M_3(p)M_6^{-1}M_5\} + \{M_2 - M_4M_6^{-1}M_5\} \quad (54)$$

The term in the last bracket and $M_3(p)M_6^{-1}M_5$ being found numerically on the digital computer. An equation similar to Equation 5 is then established and expanded to first-order differential-equation set. The rest of the analysis for the incorporation of control-equipment equations, dynamic performance, and stability limits follows as before.

This area of investigation leads to best ways for coordinating and interconnecting, control systems for dynamic stability for multi-machine operation. It is to this area which the experiences, confidences and simplifications gained with a single machine can fruitfully be extended and which are of ultimate interest and concern.

B. Two-Machine System Equations

The phasor diagram of a two-machine machine presentation is given in Figure 14. A separate reference axis is given. The generating units are described by Equation 51.

IX. REMARKS AND CONCLUSIONS

A. Machine Variables and A-constants

Machine variables and A-constants for the two values of tie-line reactances used in this study are given in Figures 15, 16, 17, and 18. Being in per-unit, as experience develops, these values will become increasingly helpful in predicting stable operation. Programs given in Appendix C.

B. Operation with Voltage Regulator

Figures 19, 20, 21, and 22 give dynamic stability with the voltage regulator only and at the nominal setting. Power output, machine damping and tie-line reactances are varied. Figure 27 summarizes these results.

Stability improves as unity power factor is approached from 0.90 power factor lagging with lighter power output and the lower tie-line reactance. This was as expected. The 0.95 lagging power factor full-load point was reported unstable at the higher tie-line reactance though close to being stable. Varying the damping had little effect on dynamic stability agreeing with Concordia's finding (6).

C. Operation with Voltage Regulator and Governor

Figures 23, 24, 25, and 26 give dynamic stability under the same conditions as section B above except with both the

governor and voltage regulator in operation. Figure 28 summarizes these results. An increase in range of stable operation to 0.975 power factor lagging was indicated.

It became evident that smaller increments of power factor should be taken at full-load output and the higher tie-line reactance. This evidenced stable operation up to and including 0.985 power factor lag, as given in Figure 30.

Lastly, it was desirable to ascertain if variations in voltage regulator settings would increase the power factor range on and above that given in Figure 29. Combinations of a few selected voltage regulator settings around the nominal setting produced results given in Figure 30. No further improvement over the nominal voltage regulator setting with the governor was evidenced. Computer time for this search was under 1.30 minutes.

D. General Observations

A derivative power signal was used for regulator damping. $dP/d\delta$ was determined from differentiation but it could have been derived through the use of A_7 and A_8 constants also.

The representation of loads under transients around an operating point needs investigation. Constant current or constant mva rather than constant impedance may be more realistic.

In a multi-machine system involving more equations,

Kron's method of matrix reduction becomes a significant practical means for reducing the number of differential equations. The present eigenvalue routine handles a 30 x 30 matrix.

The margin between this dynamical approach to stability and a real-time steady-state study versus actual field tests is an area needing study and correlation.

By a combination of successive analysis, some synthesis, and judgment based on familiarity with both the problem and analytical approach, the system design and control schemes can be adjusted to give the maximum possible stability limit. VanNess (32) developed a program giving the sensitivity of eigenvalues with changes in system parameters to determine optimal design parameters and voltage regulator settings.

The new insights gained by the author in this study of the design and operation of modern-sized generating units has been marked. It has been stimulating to attempt to align academic thinking with present practices.

The method for extending the study to a two-machine system and ultimately n-machines using the multi-machine matrix approach, with iterated load flows, is included. This work has been left for future investigation. It is hoped valid simplifications in modeling will arise in this approach to stability. These are needed to counter the

complexities associated with an expanding A-matrix resulting from increased state-space equations needed to express and interrelate a multi-machine system.

The following data reported as unstable (X) in Figure 27

0.950 pf lag, X-line = 0.718 pu, power = 1.000 pu, D = 2

0.950 pf lag, X-line = 0.718 pu, power = 1.000 pu, D = 3

0.975 pf lag, X-line = 0.414 pu, power = 1.000 pu, D = 2

are very close to stable and undoubtedly incorrect due to round-off error in finding the eigenvalues.

Figure 15. Machine variables for various armature power factors. Angles are in degrees and all other variables are in per-unit on machine rating base. Constant power output and constant 1.0 per-unit infinite bus voltage. Line reactance (X_{line}) = 0.414 pu.

Power factor	β deg	δ_b deg	V_t	V_b	δ deg	I	I_d	I_q	V_d	V_q	r_d	r_q
1.000 pu power output												
0.900 lag	- 25.84	- 17.96	1.16	1.00	56.08	0.862	0.775	0.378	0.716	0.913	0.913	- 0.717
0.925	- 22.23	- 18.32	1.14	1.01	58.75	0.853	0.759	0.391	0.739	0.868	0.868	- 0.741
0.950	- 18.19	- 18.99	1.11	1.00	62.54	0.853	0.752	0.404	0.765	0.805	0.805	- 0.766
0.975	- 12.83	- 19.94	1.07	1.00	67.96	0.863	0.753	0.420	0.795	0.716	0.716	- 0.800
1.000	0	- 22.15	0.98	1.00	82.73	0.918	0.800	0.451	0.854	0.481	0.482	- 0.555
0.975 lead	12.83	- 26.24	0.85	1.00	105.13	1.086	0.992	0.441	0.834	0.164	0.065	- 0.836
0.950	18.19	- 27.74	0.80	1.02	115.03	1.184	1.106	0.423	0.799	0.038	0.039	- 0.801
0.833 pu power output												
0.900 lag	- 25.84	- 14.95	1.15	1.01	35.23	0.725	0.634	0.351	0.663	0.939	0.940	- 0.665
0.925	- 22.23	- 15.31	1.13	1.00	37.35	0.718	0.619	0.362	0.686	0.898	0.898	- 0.687
0.950	- 18.19	- 15.64	1.11	1.01	39.89	0.711	0.604	0.376	0.712	0.852	0.852	- 0.713
0.975	- 12.83	- 16.21	1.08	1.01	43.62	0.712	0.594	0.394	0.745	0.782	0.782	- 0.746
1.000	0	- 17.94	1.00	1.00	54.85	0.750	0.613	0.432	0.818	0.576	0.577	- 0.819
0.975 lead	12.83	- 20.10	0.91	1.00	70.44	0.845	0.714	0.453	0.858	0.305	0.306	- 0.859
0.950	18.19	- 21.49	0.86	1.00	79.11	0.918	0.802	0.351	0.845	0.162	0.163	- 0.846
0.667 pu power output												
0.900 lag	- 25.84	- 12.15	1.13	1.00	31.87	0.590	0.499	0.315	0.600	0.960	0.960	- 0.598
0.925	- 22.23	- 12.25	1.12	1.00	33.43	0.579	0.479	0.326	0.617	0.935	0.935	- 0.618
0.950	- 18.19	- 12.57	1.10	1.01	35.66	0.574	0.464	0.339	0.641	0.894	0.894	- 0.642
0.975	- 12.83	- 13.11	1.07	1.00	38.99	0.575	0.452	0.355	0.673	0.832	0.832	- 0.674
1.000	0	- 13.89	1.02	1.01	47.53	0.588	0.434	0.397	0.752	0.689	0.689	- 0.753
0.975 lead	12.83	- 15.24	0.95	1.00	60.48	0.648	0.479	0.436	0.827	0.468	0.469	- 0.828
0.950	18.19	- 15.83	0.92	1.00	67.40	0.687	0.520	0.448	0.849	0.353	0.354	- 0.850

Figure 16. Machine variables for various armature power factors. Angles are in degrees and all other variables are in per-unit on machine-rating base. Constant power output and constant 1.0 per-unit infinite bus voltage. Line reactance (X_{line}) = 0.718 pu.

Power factor	θ deg	δ_b deg	V_t	V_b	δ deg	I	I_d	I_q	V_d	V_q	ψ_d	ψ_q
1.000 pu power output												
0.900 lag	- 25.84	- 29.59	1.25	1.00	65.10	0.800	0.702	0.384	0.726	1.018	1.018	- 0.727
0.925	- 22.23	- 30.75	1.21	1.00	68.96	0.804	0.700	0.395	0.748	0.951	0.951	- 0.749
0.950	- 18.19	- 32.90	1.15	1.00	75.04	0.823	0.716	0.408	0.772	0.853	0.853	- 0.773
0.975	- 12.83	- 35.99	1.07	1.00	84.00	0.863	0.753	0.420	0.795	0.716	0.716	- 0.797
1.000	0	- 49.47	0.80	1.06	118.86	1.125	1.053	0.396	0.749	0.282	0.283	- 0.751
0.975 lead	12.83	- 43.11	0.80	1.19	124.68	1.154	1.075	0.418	0.791	0.117	0.118	- 0.793
0.950	18.19	- 40.74	0.80	1.25	128.04	1.184	1.106	0.423	0.799	0.038	0.038	- 0.801
0.833 pu power output												
0.900 lag	- 25.84	- 24.12	1.25	1.00	56.37	0.667	0.566	0.352	0.667	1.057	1.058	- 0.669
0.925	- 22.23	- 25.24	1.21	1.00	59.98	0.670	0.562	0.364	0.689	0.994	0.995	- 0.691
0.950	- 18.19	- 26.30	1.17	1.00	64.02	0.718	0.559	0.378	0.716	0.925	0.926	- 0.717
0.975	- 12.83	- 28.14	1.11	1.00	70.53	0.693	0.569	0.395	0.748	0.820	0.820	- 0.749
1.000	0	- 34.95	0.93	1.00	93.60	0.806	0.689	0.420	0.794	0.484	0.485	- 0.795
0.975 lead	12.83	- 38.64	0.80	1.09	116.07	0.962	0.868	0.413	0.781	0.174	0.175	- 0.782
0.950	18.19	- 36.69	0.80	1.14	119.72	0.987	0.893	0.420	0.794	0.097	0.098	- 0.796
0.667 pu power output												
0.900 lag	- 25.84	- 19.45	1.23	1.00	48.29	0.542	0.442	0.313	0.593	1.077	1.078	- 0.594
0.925	- 22.23	- 19.76	1.21	1.00	50.23	0.536	0.427	0.324	0.614	1.043	1.043	- 0.614
0.950	- 18.19	- 20.75	1.17	1.00	53.86	0.540	0.421	0.337	0.639	0.980	0.981	- 0.640
0.975	- 12.83	- 21.65	1.13	1.00	58.14	0.544	0.413	0.355	0.672	0.908	0.909	- 0.673
1.000	0	- 24.51	1.02	1.00	72.04	0.588	0.411	0.397	0.752	0.689	0.689	- 0.753
0.975 lead	12.83	- 30.97	0.84	1.00	99.51	0.733	0.605	0.413	0.781	0.307	0.308	- 0.783
0.950	18.19	- 31.77	0.80	1.04	108.56	0.789	0.674	0.411	0.779	0.183	0.183	- 0.780

Figure 17. Synchronous generator A-constants and operating angles δ in degrees versus various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. Line reactance (X-line) = 0.414 pu.

Power factor	δ deg	A1	A2	A3	A4	A5	A6	A7	A8
1.000 pu power output									
0.900 lag	56.08	- 1.263	- 1.210	- 1.750	- 3.121	- 0.006	0.436	1.104	1.190
0.925	58.75	- 1.252	- 1.246	- 1.810	- 3.121	- 0.012	0.422	1.125	1.176
0.950	62.54	- 1.228	- 1.287	- 1.877	- 3.121	- 0.025	0.402	1.145	1.165
0.975	67.96	- 1.193	- 1.335	- 1.957	- 3.121	- 0.047	0.371	1.165	1.154
1.000	82.73	- 1.081	- 1.428	- 2.109	- 3.121	- 0.127	0.272	1.194	1.151
0.975 lead	105.13	- 0.697	- 1.397	- 2.060	- 3.121	- 0.295	0.106	1.148	1.211
0.950	115.03	- 0.455	- 1.342	- 1.970	- 3.121	- 0.373	0.026	1.100	1.240
0.833 pu power output									
0.900 lag	50.18	- 1.186	- 1.120	- 1.622	- 3.121	- 0.026	0.453	1.037	1.157
0.925	52.66	- 1.175	- 1.154	- 1.680	- 3.121	- 0.023	0.440	1.055	1.140
0.950	55.53	- 1.169	- 1.196	- 1.748	- 3.121	- 0.018	0.425	1.074	1.120
0.975	59.83	- 1.158	- 1.249	- 1.835	- 3.121	- 0.007	0.401	1.094	1.099
1.000	72.78	- 1.113	- 1.364	- 2.024	- 3.121	- 0.046	0.319	1.127	1.077
0.975 lead	90.54	- 0.972	- 1.428	- 2.128	- 3.121	- 0.157	0.185	1.140	1.114
0.950	100.63	- 0.815	- 1.408	- 2.095	- 3.121	- 0.232	0.104	1.125	1.155
0.667 pu power output									
0.900 lag	44.018	- 1.070	- 1.006	- 1.458	- 3.121	- 0.051	0.471	0.951	1.116
0.925	45.676	- 1.070	- 1.038	- 1.512	- 3.121	- 0.053	0.462	0.967	1.090
0.950	48.230	- 1.063	- 1.077	- 1.575	- 3.121	- 0.051	0.450	0.981	1.063
0.975	52.093	- 1.053	- 1.127	- 1.658	- 3.121	- 0.046	0.431	0.995	1.034
1.000	61.415	- 1.068	- 1.253	- 1.863	- 3.121	- 0.027	0.374	1.018	0.967
0.975 lead	75.719	- 1.055	- 1.372	- 2.056	- 3.121	- 0.037	0.273	1.042	0.969
0.950	83.230	- 1.026	- 1.409	- 2.115	- 3.121	- 0.081	0.213	1.057	0.994

Figure 18. Synchronous generator A-constants and operating angles δ in degrees versus various armature power factors. Constant power output and constant 1.0 per unit infinite bus voltage. Line reactance (X-line) = 0.718 pu.

Power factor	δ deg	A1	A2	A3	A4	A5	A6	A7	A8
1.000 pu power output									
0.900 lag	65.10	- 0.931	- 0.996	- 1.357	- 2.507	- 0.055	0.556	0.837	0.810
0.925	68.96	- 0.901	- 1.025	- 1.404	- 2.507	- 0.070	0.537	0.847	0.803
0.950	75.04	- 0.841	- 1.054	- 1.452	- 2.507	- 0.101	0.506	0.854	0.801
0.975	84.00	- 0.747	- 1.084	- 1.501	- 2.507	- 0.155	0.457	0.856	0.806
1.000	118.86	- 0.145	- 1.026	- 1.410	- 2.507	- 0.451	0.240	0.770	0.872
0.975 lead	124.68	- 0.097	- 1.079	- 1.498	- 2.507	- 0.533	0.100	0.793	0.868
0.950	128.04	- 0.037	- 1.089	- 1.515	- 2.507	- 0.576	0.032	0.794	0.870
0.833 pu power output									
0.900 lag	56.37	- 0.920	- 0.915	- 1.247	- 2.507	- 0.007	0.578	0.788	0.780
0.925	59.98	- 0.892	- 0.943	- 1.293	- 2.507	- 0.017	0.561	0.800	0.771
0.950	64.02	- 0.869	- 0.976	- 1.347	- 2.507	- 0.029	0.540	0.807	0.759
0.975	70.53	- 0.827	- 1.017	- 1.414	- 2.507	- 0.057	0.504	0.816	0.752
1.000	93.60	- 0.614	- 1.076	- 1.510	- 2.507	- 0.204	0.355	0.811	0.786
0.975 lead	116.07	- 0.304	- 1.059	- 1.486	- 2.507	- 0.406	0.148	0.776	0.837
0.950	119.72	- 0.271	- 1.076	- 1.513	- 2.507	- 0.447	0.083	0.780	0.839
0.667 pu power output									
0.900 lag	48.29	- 0.856	- 0.814	- 1.107	- 2.507	- 0.028	0.598	0.725	0.747
0.925	50.23	- 0.850	- 0.840	- 1.149	- 2.507	- 0.028	0.589	0.735	0.727
0.950	53.86	- 0.828	- 0.871	- 1.201	- 2.507	- 0.022	0.572	0.742	0.711
0.975	58.14	- 0.813	- 0.913	- 1.269	- 2.507	- 0.013	0.549	0.749	0.688
1.000	72.04	- 0.767	- 1.015	- 1.433	- 2.507	- 0.038	0.461	0.759	0.667
0.975 lead	99.51	- 0.543	- 1.053	- 1.496	- 2.507	- 0.227	0.250	0.754	0.757
0.950	108.56	- 0.447	- 1.049	- 1.490	- 2.507	- 0.305	0.156	0.746	0.785

Figure 19. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.414 pu. Machine damping $D = 2$.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag	0.925	0.950	0.975	1.000	0.975 lead	0.950
Power output = 1.000 pu						
(Stable)	(Stable)	(Stable)	(Stable)	(Stable)		
- 27.063 - 196.716 - 26.894 - 196.695 - 26.856 - 196.691 - 26.826 - 196.688 - 26.789 - 196.683 - 26.783 - 196.675 - 26.780 - 196.713						
- 27.063 196.716 - 26.894 196.695 - 26.856 196.691 - 26.826 196.688 - 26.789 196.683 - 26.783 196.675 - 26.780 196.713						
- 145.595 0 - 145.674 0 - 145.686 0 - 145.704 0 - 145.760 0 - 145.954 0 - 145.980 0						
- 1.094 - 0.407 - 0.872 0 - 0.932 0 - 1.074 - 0.165 - 0.871 0 - 0.868 0 - 0.965 0						
- 1.094 0.407 - 1.242 0 - 1.201 0 - 1.074 0.165 - 1.288 0 - 1.195 0 - 1.096 0						
- 0.006 0 - 0.005 0 - 0.015 0 - 0.017 0 - 0.0176 - 0.039 - 0.019 0 - 0.016 0						
- 0.164 0 - 0.159 0 - 0.120 0 - 0.085 0 - 0.0176 - 0.039 0.081 0 - 0.101 0						

Power output = 0.833 pu

(Stable)	(Stable)	(Stable)	(Stable)	(Stable)	(Stable)	(Stable)
- 26.877 - 196.694 - 26.849 - 196.690 - 26.827 - 196.688 - 26.804 - 196.685 - 26.773 - 196.681 - 26.760 - 196.680 - 26.760 - 196.662						
- 26.877 196.694 - 26.849 196.690 - 26.827 196.688 - 26.804 196.685 - 26.773 196.681 - 26.760 196.680 - 26.760 196.662						
- 145.708 0 - 145.714 0 - 145.716 0 - 145.722 0 - 145.745 0 - 145.815 0 - 145.894 0						
- 0.805 0 - 0.766 0 - 0.935 0 - 0.909 0 - 1.072 - 0.051 - 0.941 0 - 0.893 0						
- 1.241 0 - 1.286 0 - 1.148 0 - 1.193 0 - 1.072 0.051 - 1.197 0 - 1.202 0						
- 0.004 0 - 0.002 0 - 0.010 0 - 0.014 0 - 0.033 - 0.026 - 0.001 - 0.035 - 0.012 0						
- 1.195 0 - 0.187 0 - 0.146 0 - 0.117 0 - 0.033 0.026 - 0.001 0.035 0.047 0						

Power output = 0.667 pu

(Stable)	(Stable)	(Stable)	(Stable)	(Stable)	(Stable)	(Stable)
- 26.830 - 196.688 - 26.816 - 196.686 - 26.799 - 196.683 - 26.783 - 196.682 - 26.760 - 196.679 - 26.740 - 196.773 - 26.740 - 196.670						
- 26.830 196.688 - 26.816 196.686 - 26.799 196.683 - 26.783 196.682 - 26.760 196.679 - 26.740 196.773 - 26.740 196.670						
- 145.767 0 - 145.767 0 - 145.771 0 - 145.776 0 - 145.768 0 - 145.774 0 - 145.787 0						
- 0.820 0 - 0.815 0 - 0.788 0 - 0.808 0 - 0.870 0 - 0.932 0 - 0.965 0						
- 1.155 0 - 1.174 0 - 1.218 0 - 1.217 0 - 1.218 0 - 1.201 0 - 1.177 0						
- 0.003 0 - 0.002 0 - 0.008 0 - 0.005 0 - 0.021 0 - 0.005 0 - 0.007 0						
- 0.210 0 - 0.197 0 - 0.171 0 - 0.150 0 - 0.079 0 - 0.042 0 - 0.017 0						

Figure 20. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.414 pu. Machine damping $D = 3$.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag	0.925	0.950	0.975	1.000	0.975 lead	0.950
-----------	-------	-------	-------	-------	------------	-------

Power output = 1.000 pu

(Stable)	(Stable)	(Stable)	(Stable)
----------	----------	----------	----------

-	26.935	-	196.700	-	26.894	-	196.696	-	26.857	-	196.692	-	26.825	-	196.687	-	26.789	-	196.683	-	26.783	-	196.682	-	26.781	-	196.683	0
-	26.935		196.700	-	26.894	-	196.696	-	26.857	-	196.692	-	26.825	-	196.687	-	26.789	-	196.683	-	26.783	-	196.682	-	26.781	-	196.683	0
-	219.		0	-	219.038		0	-	219.046		0	-	219.058		0	-	219.096		0	-	219.224		0	-	219.241		0	0
-	0.847		0	-	0.790		0	-	0.779		0	-	0.784		0	-	0.775		0	-	0.794		0	-	0.847		0	0
-	1.083		0	-	1.147		0	-	1.171		0	-	1.176		0	-	1.209		0	-	1.137		0	-	1.085		0	0
-	0.003		0	-	0.003		0	-	0.008		0	-	0.001		0	-	0.014	-	0.036	-	0.013		0	-	0.014		0	0
-	0.139		0	-	0.128		0	-	0.102		0	-	0.089		0	-	0.014		0.036	-	0.061		0	-	0.080		0	0

Power output = 0.833 pu

(Stable)	(Stable)	(Stable)	(Stable)	(Stable)
----------	----------	----------	----------	----------

-	26.877	-	196.693	-	26.849	-	196.689	-	26.827	-	196.687	-	26.805	-	196.684	-	26.773	-	196.681	-	26.760	-	196.679	-	26.760	-	196.680
-	26.877		196.693	-	26.849		196.689	-	26.827		196.687	-	26.805		196.684	-	26.773		196.681	-	26.760		196.679	-	26.760		196.680
-	219.061	0	-	219.065	0	-	219.067	0	-	219.070	0	-	219.085	0	-	219.132	0	-	219.185	0							
-	0.817	0	-	0.798	0	-	0.778	0	-	0.860	0	-	0.841	0	-	0.867	0	-	0.859	0							
-	1.078	0	-	1.107	0	-	1.138	0	-	1.077	0	-	1.124	0	-	1.104	0	-	1.087	0							
-	0.008	0	-	0.009	0	-	0.011	0	-	0.011	0	-	0.010	0	-	0.010	0	-	0.026	0							
-	0.144	0	-	0.129	0	-	0.114	0	-	0.091	0	-	0.056	0	-	0.014	0	-	0.049	0							

Power output = 0.667 pu

(Stable)	(Stable)	(Stable)	(Stable)	(Stable)	(Stable)	(Stable)
----------	----------	----------	----------	----------	----------	----------

- 26.830	- 196.688	- 26.816	- 196.687	- 26.800	- 196.684	- 26.783	- 196.682	- 26.760	- 196.680	- 26.745	- 196.678	- 26.741	- 196.677
- 26.830	196.688	- 26.816	196.687	- 26.800	196.684	- 26.783	196.682	- 26.760	196.680	- 26.745	196.678	- 26.741	196.677
- 219.100	0	- 219.099	0	- 219.102	0	- 219.105	0	- 219.100	0	- 219.103	0	- 219.114	0
- 0.930	- 0.119	- 0.932	- 0.091	- 0.938	- 0.093	- 0.946	- 0.113	- 0.796	0	- 0.819	0	- 1.013	0
- 0.930	0.119	- 0.932	0.091	- 0.938	0.093	- 0.946	0.113	- 1.131	0	- 1.144	0	- 0.009	- 0.011
- 0.009	0	- 0.005	0	- 0.007	0	- 0.012	0	- 0.016	0	- 0.005	0	- 0.009	- 0.011
- 0.140	0	- 0.137	0	- 0.122	0	- 0.098	0	- 0.063	0	- 0.034	0		

Figure 21. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) ≈ 0.718 pu. Machine damping $D = 2$.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 27.063	- 196.716	- 26.954	- 196.703	- 26.880	- 196.693	- 26.826	- 196.687	- 26.817	- 196.685	- 26.809	- 196.685	- 26.807	- 196.677
- 27.063	196.716	- 26.954	196.703	- 26.880	196.693	- 26.826	196.687	- 26.817	196.685	- 26.809	196.685	- 26.807	- 196.677
- 145.84	0	- 145.852	0	- 145.881	0	- 145.929	0	- 146.414	0	- 146.457	0	- 146.500	0
- 0.952	0	- 0.950	- 0.387	- 0.947	0	- 0.760	0	- 0.680	0	- 0.680	0	- 0.662	0
- 0.952	0	- 0.950	- 0.387	- 0.947	0	- 1.102	0	- 0.993	0	- 0.996	0	- 1.005	0
- 0.007	0	- 0.004	0	0.002	- 0.440	- 0.017	- 0.031	0.007	0	0.011	0	- 0.003	0
- 0.080	0	- 0.073	0	- 0.055	0.440	- 0.017	0.031	0.253	0	0.297	0	0.344	0

Power output = 0.833 pu

(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 26.979	- 196.705	- 26.902	- 196.700	- 26.85	- 196.69	- 26.811	- 196.685	- 26.772	- 196.681	- 26.781	- 196.681	- 26.781	- 196.682
- 26.979	196.705	- 26.902	196.700	- 26.85	196.69	- 26.811	196.685	- 26.772	196.681	- 26.781	196.681	- 26.781	196.682
- 145.841	0	- 145.856	0	- 145.87	0	- 145.890	0	- 145.996	0	- 146.316	0	- 146.347	0
- 0.632	0	- 0.750	0	- 0.747	0	- 0.729	0	- 0.782	0	- 0.677	0	- 0.701	0
- 1.180	0	- 1.088	0	- 1.10	0	- 1.123	0	- 1.059	0	- 1.026	0	- 0.996	0
- 0.001	0	- 0.016	0	- 0.019	0	- 0.009	0	0.005	- 0.032	0.001	0	0.007	0
- 0.172	0	- 0.117	0	- 0.091	0	- 0.078	0	0.005	0.032	0.192	0	0.207	0

Power output = 0.667 pu

(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 26.895	- 196.696	- 26.861	- 196.692	- 26.824	- 196.688	- 26.796	- 196.684	- 26.760	- 196.680	- 26.747	- 196.677	- 26.754	- 196.678
- 26.895	196.696	- 26.861	- 196.692	- 26.824	196.688	- 26.796	196.684	- 26.760	196.680	- 26.747	196.677	- 26.754	196.678
- 145.875	0	- 145.878	0	- 145.889	0	- 145.897	0	- 145.919	0	- 146.032	0	- 146.218	0
- 0.623	0	- 0.619	0	- 0.639	0	- 0.750	0	- 0.697	0	- 0.722	0	- 0.729	0
- 1.143	0	- 1.147	0	- 1.153	0	- 1.067	0	- 1.147	0	- 1.100	0	- 0.993	0
- 0.008	0	- 0.011	0	- 0.010	0	- 0.014	- 0.019	- 0.032	- 0.019	0.012	- 0.029	- 0.002	0
- 0.179	0	- 0.160	0	- 0.136	0	- 0.101	- 0.019	- 0.032	0.019	0.012	0.029	- 0.116	0

Figure 22. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu. Machine damping $D = 3$.

Real plus imaginary parts of eigenvalues vs power factors														
0.900 lag			0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu														
(Stable)			(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 27.063	- 196.716	- 26.954	- 196.703	- 26.875	- 196.694	- 26.825	- 196.687	- 26.817	- 196.686	- 26.808	- 196.686	- 26.810	- 196.706	- 196.706
- 27.063	196.716	- 26.954	196.703	- 26.875	196.694	- 26.825	196.687	- 26.817	196.686	- 26.808	196.686	- 26.810	196.706	196.706
- 219.146	0	- 219.156	0	- 219.176	0	- 219.208	0	- 219.530	0	- 219.559	0	- 219.588	0	0
- 0.637	0	- 0.696	0	- 0.633	0	- 0.680	0	- 0.631	0	- 0.638	0	- 0.642	0	0
- 1.094	0	- 1.045	0	- 1.109	0	- 1.067	0	- 1.000	0	- 1.000	0	- 0.989	0	0
- 0.002	0	- 0.004	0	- 0.001	0	- 0.013	0	- 0.001	0	- 0.004	0	- 0.001	0	0
- 0.101	0	- 0.079	0	- 0.063	0	- 0.013	0	- 0.181	0	- 0.213	0	- 0.240	0	0
Power output = 0.833 pu														
(Stable)			(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 26.980	- 196.707	- 26.903	- 196.697	- 26.852	- 196.691	- 26.812	- 196.686	- 26.772	- 196.680	- 26.782	- 196.682	- 26.780	- 196.684	- 196.684
- 26.980	196.707	- 26.903	196.697	- 26.852	196.691	- 26.812	196.686	- 26.772	196.680	- 26.782	196.682	- 26.780	196.684	196.684
- 219.150	0	- 219.159	0	- 219.167	0	- 219.181	0	- 219.252	0	- 219.466	0	- 219.484	0	0
- 0.617	0	- 0.622	0	- 0.697	0	- 0.632	0	- 0.767	0	- 0.658	0	- 0.653	0	0
- 1.089	0	- 1.095	0	- 1.035	0	- 1.101	0	- 0.969	0	- 0.989	0	- 1.000	0	0
- 0.004	0	- 0.028	0	- 0.041	0	- 0.033	0.023	- 0.004	0.043	- 0.019	0	- 0.033	0	0
- 0.119	0	- 0.076	0	- 0.041	0	- 0.033	0.023	- 0.004	0.043	- 0.153	0	- 0.125	0	0
Power output = 0.667 pu														
(Stable)			(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 26.986	- 196.697	- 26.862	- 196.692	- 26.824	- 196.688	- 26.797	- 196.684	- 26.760	- 196.680	- 26.747	- 196.678	- 26.756	- 196.680	- 196.680
- 26.986	196.697	- 26.862	196.692	- 26.824	196.688	- 26.797	196.684	- 26.760	196.680	- 26.747	196.678	- 26.756	196.680	196.680
- 219.171	0	- 219.173	0	- 219.181	0	- 219.186	0	- 219.201	0	- 219.276	0	- 219.401	0	0
- 0.714	0	- 0.715	0	- 0.743	0	- 0.720	0	- 0.787	0	- 0.715	0	- 0.664	0	0
- 0.967	0	- 0.974	0	- 0.954	0	- 0.988	0	- 0.948	0	- 1.008	0	- 0.998	0	0
- 0.008	0	- 0.018	0	- 0.012	0	- 0.043	0	- 0.022	0.037	- 0.010	- 0.035	- 0.005	0	0
- 0.120	0	- 0.100	0	- 0.091	0	- 0.043	0	- 0.022	0.037	- 0.010	- 0.035	- 0.087	0	0

Figure 23. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.414 pu. Machine damping $D = 2$.

Real plus imaginary parts of eigenvalues vs power factors													
0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)					
- 0.100	0	- 0.100	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 6.667	0	- 6.667	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 10.000	0	- 10.000	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 26.934	- 196.706	- 26.329	- 196.698	- 26.856	- 196.693	- 26.325	- 196.691	- 26.790	- 196.687	- 26.783	- 196.687	- 26.788	- 196.688
- 26.934	196.706	- 26.329	196.698	- 26.856	196.693	- 26.325	196.691	- 26.790	196.687	- 26.783	196.687	- 26.788	196.688
- 145.669	0	- 145.676	0	- 145.686	0	- 145.703	0	- 145.760	0	- 145.953	0	- 146.074	0
- 0.999	0	- 1.000	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 1.110	0	- 1.123	0	- 1.134	0	- 1.146	0	- 1.158	0	- 1.066	0	- 1.002	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.010	0	- 0.001	0	- 0.002	0
- 0.170	0	- 0.152	0	- 0.129	0	- 0.100	0	- 0.026	0	- 0.066	0	- 0.125	0
Power output = 0.833 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)			
- 0.100	0	- 0.100	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 6.667	0	- 6.667	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.1000	0	- 0.100	0
- 10.000	0	- 10.000	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 26.876	- 196.696	- 26.849	- 196.694	- 26.827	- 196.694	- 26.804	- 196.690	- 26.773	- 196.684	- 26.761	- 196.679	- 26.759	- 196.684
- 26.876	196.696	- 26.849	196.694	- 26.827	196.694	- 26.804	196.690	- 26.773	196.684	- 26.761	196.679	- 26.759	196.684
- 145.708	0	- 145.713	0	- 145.716	0	- 145.721	0	- 145.744	0	- 145.814	0	- 145.894	0
- 0.999	0	- 0.999	0	- 0.999	0	- 1.000	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 1.057	0	- 1.068	0	- 1.085	0	- 1.104	0	- 1.141	0	- 1.136	0	- 1.100	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.006	0	- 0.001	- 0.012	- 0.001	0
- 0.185	0	- 0.165	0	- 0.149	0	- 0.124	0	- 0.063	0	- 0.001	0.012	0.034	0
Power output = 0.667 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 0.100	0	- 0.100	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 6.667	0	- 6.667	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 10.000	0	- 10.000	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0	- 6.667	0
- 26.828	- 196.692	- 26.815	- 196.690	- 26.798	- 196.689	- 26.783	- 196.685	- 26.758	- 196.681	- 26.744	- 196.682	- 26.741	- 196.680
- 26.828	196.692	- 26.815	196.690	- 26.798	196.689	- 26.783	196.685	- 26.758	196.681	- 26.744	196.682	- 26.741	196.680
- 145.766	0	- 145.766	0	- 145.769	0	- 145.774	0	- 145.767	0	- 145.774	0	- 145.788	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 1.000	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.987	0	- 0.999	0	- 1.014	0	- 1.033	0	- 1.088	0	- 1.134	0	- 1.142	0
- 0.004	0	- 0.004	0	- 0.004	0	- 0.004	0	- 0.005	0	- 0.006	0	- 0.012	- 0.006
- 0.197	0	- 0.184	0	- 0.166	0	- 0.142	0	- 0.094	0	- 0.041	0	- 0.012	0.006

Figure 24. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.414 pu. Machine damping $D = 3$.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag		0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.000 pu													
(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.933	- 196.700	- 26.392	- 196.697	- 26.845	- 196.692	- 26.824	- 196.689	- 26.788	- 196.685	- 26.783	- 196.682	- 26.787	- 196.684
- 26.933	196.700	- 26.392	196.697	- 26.845	196.692	- 26.824	196.689	- 26.788	196.685	- 26.783	196.682	- 26.787	196.684
- 219.034	0	- 219.039	0	- 219.047	0	- 219.057	0	- 219.095	0	- 219.223	0	- 219.034	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.933	0	- 0.943	0	- 0.954	0	- 0.965	0	- 0.982	0	- 0.932	0	- 0.993	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.013	0	- 0.001	0	- 0.002	0
- 0.134	0	- 0.120	0	- 0.102	0	- 0.078	0	- 0.016	0	- 0.050	0	- 0.093	0

Power output = 0.833 pu

(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.875	- 196.694	- 26.848	- 196.691	- 26.825	- 196.691	- 26.803	- 196.686	- 26.772	- 196.679	- 26.760	- 196.679	- 26.758	- 196.682
- 26.375	196.694	- 26.848	196.691	- 26.825	196.691	- 26.803	196.686	- 26.772	196.679	- 26.760	196.679	- 26.758	196.682
- 219.061	0	- 219.065	0	- 219.067	0	- 219.070	0	- 219.084	0	- 219.131	0	- 219.184	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.897	0	- 0.907	0	- 0.920	0	- 0.935	0	- 0.957	0	- 0.972	0	- 0.948	0
- 0.005	0	- 0.005	0	- 0.005	0	- 0.005	0	- 0.006	0	- 0.001	0.011	- 0.001	0
- 0.145	0	- 0.131	0	- 0.117	0	- 0.097	0	- 0.049	0	- 0.001	0.011	- 0.026	0

Power output = 0.667 pu

(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)		(Stable)	
- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0	- 0.100	0
- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0	- 5.567	0
- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0	- 10.000	0
- 26.827	- 196.690	- 26.814	- 196.688	- 26.793	- 196.686	- 26.781	- 196.683	- 26.758	- 196.678	- 26.743	- 196.677	- 26.740	- 196.677
- 26.827	196.690	- 26.814	196.688	- 26.798	196.686	- 26.781	196.683	- 26.758	196.678	- 26.743	196.677	- 26.740	196.677
- 219.100	0	- 219.101	0	- 219.102	0	- 219.105	0	- 219.111	0	- 219.104	0	- 219.113	0
- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0	- 0.999	0
- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0	- 19.999	0
- 0.851	0	- 0.861	0	- 0.873	0	- 0.887	0	- 0.929	0	- 0.965	0	- 0.973	0
- 0.004	0	- 0.004	0	- 0.004	0	- 0.004	0	- 0.005	0	- 0.006	0	- 0.010	- 0.007
- 0.151	0	- 0.141	0	- 0.128	0	- 0.110	0	- 0.073	0	- 0.031	0	- 0.010	0.007

Figure 25. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu. Machine damping $D = 2$.

Real plus imaginary parts of eigenvalues vs power factors

0.900 lag	0.925	0.950	0.975	1.000	0.975 lead	0.950
Power output = 1.000 pu						
(Stable)	(Stable)	(Stable)	(Stable)			
- 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000
- 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100
- 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667
- 27.063	- 196.722 - 26.934	- 196.707 - 26.876	- 196.690 - 26.826	- 196.686 - 26.801	- 196.686 - 26.790	- 196.687 - 26.787
- 27.063	196.722 - 26.934	196.707 - 26.876	196.690 - 26.826	196.686 - 26.801	196.686 - 26.790	196.687 - 26.787
- 145.837	0 - 145.851	0 - 145.811	0 - 145.928	0 - 146.231	0 - 146.254	0 - 146.284
- 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999
- 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999
- 0.869	0 - 0.874	0 - 0.872	0 - 0.865	0 - 0.865	0 - 0.757	0 - 0.753
- 0.005	0 - 0.006	0 - 0.007	0 - 0.017	0 - 0.007	0 - 0.002	0 - 0.002
- 0.116	0 - 0.096	0 - 0.066	0 - 0.017	0.007 - 0.149	0 - 0.186	0 - 0.212

Power output = 0.833 pu

(Stable)	(Stable)	(Stable)	(Stable)			
- 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100
- 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667
- 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000
- 26.978	- 196.711 - 26.901	- 196.703 - 26.851	- 196.694 - 26.810	- 196.691 - 26.771	- 196.684 - 26.768	- 196.686 - 26.765
- 26.978	196.711 - 26.901	196.703 - 26.851	196.694 - 26.810	196.691 - 26.771	196.684 - 26.768	196.686 - 26.765
- 145.842	0 - 145.836	0 - 145.867	0 - 145.889	0 - 145.995	0 - 146.151	0 - 145.842
- 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999
- 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999
- 0.837	0 - 0.841	0 - 0.850	0 - 0.857	0 - 0.838	0 - 0.780	0 - 0.781
- 0.005	0 - 0.005	0 - 0.006	0 - 0.006	0 - 0.004	0.012 - 0.001	0 - 0.001
- 0.143	0 - 0.124	0 - 0.104	0 - 0.074	0 - 0.004	0.012 - 0.106	0 - 0.123

Power output = 0.667 pu

(Stable)	(Stable)	(Stable)	(Stable)	(Stable)		
- 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100	0 - 0.100
- 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667	0 - 6.667
- 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000	0 - 10.000
- 26.894	- 196.699 - 26.681	- 196.695 - 26.823	- 196.691 - 26.796	- 196.689 - 26.759	- 196.683 - 26.746	- 196.682 - 26.745
- 26.894	196.699 - 26.681	196.695 - 26.823	196.691 - 26.796	196.689 - 26.759	196.683 - 26.746	196.682 - 26.745
- 145.875	0 - 145.877	0 - 145.889	0 - 145.889	0 - 145.889	0 - 145.889	0 - 145.889
- 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999	0 - 0.999
- 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999	0 - 19.999
- 0.783	0 - 0.792	0 - 0.801	0 - 0.815	0 - 0.847	0 - 0.819	0 - 0.800
- 0.005	0 - 0.005	0 - 0.005	0 - 0.005	0 - 0.006	0.004	0 - 0.001
- 0.164	0 - 0.152	0 - 0.133	0 - 0.111	0 - 0.054	0.019	0 - 0.054

Figure 26. Eigenvalue determination of dynamic stability for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu. Machine damping $D = 3$.

Real plus imaginary parts of eigenvalues vs power factors												
0.900 lag	0.925		0.950		0.975		1.000		0.975 lead		0.950	
Power output = 1.006 pu												
(Stable)	(Stable)		(Stable)		(Stable)							
- 10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000
- 0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100
- 6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667
- 27.061	- 196.717	- 26.952	- 196.705	- 26.874	- 196.693	- 26.825	- 196.686	- 26.800	- 196.684	- 26.790	- 196.682	- 26.787
- 27.061	196.717	- 26.952	196.705	- 26.874	196.693	- 26.825	196.686	- 26.800	196.684	- 26.790	196.682	- 26.787
- 219.147	0 -	219.157	0 -	219.157	0 -	219.207	0 -	219.109	0 -	219.124	0 -	219.124
- 0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999
- 19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999
- 0.738	0 -	0.743	0 -	0.746	0 -	0.745	0 -	0.678	0 -	0.689	0 -	0.688
- 0.006	0 -	0.005	0 -	0.008	0 -	0.013	- 0.009	- 0.002	0 -	0.002	0 -	0.002
- 0.091	0 -	0.074	0 -	0.050	0 -	0.013	0.009	0.109	0 -	0.136	0 -	0.155
Power output = 0.833 pu												
(Stable)	(Stable)		(Stable)		(Stable)							
- 0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100
- 6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667
- 10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000
- 26.976	- 196.709	- 26.900	- 196.700	- 26.850	- 196.691	- 26.810	- 196.686	- 26.771	- 196.680	- 26.767	- 196.683	- 26.764
- 26.976	196.709	- 26.900	196.700	- 26.850	196.691	- 26.810	196.686	- 26.771	196.680	- 26.767	196.683	- 26.764
- 219.151	0 -	219.160	0 -	219.167	0 -	219.182	0 -	219.232	0 -	219.356	0 -	219.367
- 0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999
- 19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999
- 0.713	0 -	0.719	0 -	0.727	0 -	0.735	0 -	0.732	0 -	0.701	0 -	0.703
- 0.005	0 -	0.006	0 -	0.006	0 -	0.006	0 -	0.002	- 0.011	- 0.001	0 -	0.001
- 0.112	0 -	0.096	0 -	0.080	0 -	0.057	0 -	0.002	- 0.001	- 0.078	0 -	0.091
Power output = 0.667 pu												
(Stable)	(Stable)		(Stable)		(Stable)		(Stable)		(Stable)			
- 0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100	0 -	0.100
- 6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667	0 -	6.667
- 10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000	0 -	10.000
- 26.893	- 196.696	- 26.860	- 196.695	- 26.822	- 196.687	- 26.795	- 196.687	- 26.758	- 196.683	- 26.745	- 196.677	- 26.745
- 26.893	196.696	- 26.860	196.695	- 26.822	196.687	- 26.795	196.687	- 26.758	196.683	- 26.745	196.677	- 26.745
- 219.173	0 -	219.174	0 -	219.181	0 -	219.186	0 -	219.202	0 -	219.277	0 -	219.305
- 0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999	0 -	0.999
- 19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999	0 -	19.999
- 0.667	0 -	0.685	0 -	0.692	0 -	0.704	0 -	0.731	0 -	0.721	0 -	0.661
- 0.005	0 -	0.005	0 -	0.005	0 -	0.005	0 -	0.007	0 -	0.004	0 -	0.711
- 0.126	0 -	0.117	0 -	0.102	0 -	0.085	0 -	0.041	0 -	0.013	0 -	0.040

Line reactance damping	X-line = 0.414 pu						X-line = 0.718 pu					
	D = 2			D = 3			D = 2			D = 3		
	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667
power output, pu power factor												
0.900 lag	S	S	S	S	S	S	S	S	S	S	S	S
0.925	S	S	S	S	S	S	S	S	S	S	S	S
0.950	S	S	S	S	S	S	X	S	S	X	S	S
0.975	S	S	S	X	S	S	S	S	S	S	S	S
1.000	S	S	S	S	S	S	X	X	S	X	X	S
0.975 lead	X	S	S	X	X	S	X	X	X	X	X	X
0.950	X	X	S	X	X	S	X	X	X	X	X	X

Figure 27. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With automatic voltage regulator only set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$).

Line reactance damping power output, pu power factor	X-line = 0.414 pu						X-line = 0.718 pu					
	D = 2			D = 3			D = 2			D = 3		
	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667	1.000	0.833	0.667
0.900 lag	S	S	S	S	S	S	S	S	S	S	S	S
0.925	S	S	S	S	S	S	S	S	S	S	S	S
0.950	S	S	S	S	S	S	S	S	S	S	S	S
0.975	S	S	S	S	S	S	S	S	S	S	S	S
1.000	S	S	S	S	S	S	X	X	S	X	X	S
0.975 lead	X	S	S	X	S	S	X	X	X	X	X	X
0.950	X	X	S	X	X	S	X	X	X	X	X	X

Figure 28. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$).

Machine damping power output, pu power factor	D = 2			D = 3		
	1.000	0.833	0.667	1.000	0.833	0.667
0.97 lag	S	S	S	S	S	S
0.98	S	S	S	S	S	S
0.99	X	S	S	X	S	S
1.00	X	X	S	X	X	S
0.99 lead	X	X	S	X	X	S
0.98	X	X	X	X	X	X

Figure 29. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output and constant 1.0 per-unit infinite bus voltage. With governor and automatic voltage regulator set at nominal values ($K_A = 400$, $K_F = 0.06$, $K_D = 4$, $T_D = 1$). Line reactance (X-line) = 0.718 pu only and finer power factor increments from 0.970 lag.

Voltage regulator variables								
T_D	K_D	K_F	K_A	0.980 Lag	0.985	0.990	0.995	1.000
0.8	1	0.04	200	S	S	X	X	X
1.0	4	0.06	400	(All combinations of voltage regulator settings to the left resulted in stability determination as above.)				
1.2	7	0.08	600					

Figure 30. Summary of eigenvalue determination of dynamic stability (S = stable; X = unstable) for various armature power factors. Constant power output of 1.0 per-unit infinite bus voltage. With governor. Automatic voltage regulator set at various settings near nominal to scan for optimal regulator setting. Machine damping $D = 2$. Line reactance (X-line) = 0.718 pu.

X. LITERATURE CITED

1. Anderson, P. M. and Fouad, A. A. Power system control and stability. Unpublished notes. Electrical Engineering Department, Iowa State University, Ames, Iowa. 1968.
2. Brown, R. G. Elements of linear state-space analysis. Unpublished notes. Electrical Engineering Department, Iowa State University, Ames, Iowa. 1966.
3. Brown, R. G. and Nilsson, J. W. Introduction to linear system analysis. John Wiley and Sons, Inc., New York, New York. 1962.
4. Byerly, R. T., Skooglund, J. W. and Keay, F. W. Control of generator excitation for improved power system stability. American Power Conference Proceedings 1967: 1011-1022. 1967.
5. Coddington, E. A. and Levison, Norman. Theory of ordinary differential equations. McGraw Hill Book Co., Inc., New York, New York. 1955.
6. Concordia, C. Modern concepts of power system stability. Institute of Electrical Engineers of Japan Journal 85: 4-12. 1965.
7. Concordia, C. Steady-state stability of synchronous machines as affected by voltage regulator characteristics. American Institute of Electrical Engineers Transactions 63, Part 3: 215-220. 1944.
8. Concordia, C. Synchronous machines. John Wiley and Sons, Inc., New York, New York. 1951.
9. Crary, S. B. Power system stability. Vols. 1 and 2. John Wiley and Sons, Inc., New York, New York. 1945.
10. Doherty, R. E. and Nickle, C. A. Synchronous machines. American Institute of Electrical Engineers Transactions 47: 912-947. 1926.
11. Ewart, D. N., Landgren, G. L., Temoshok, M. and Walkey, W. W. Stability studies and tests on a 532 mw cross-compound turbine-generator set. Institute of Electrical and Electronics Engineers Transactions on Power Apparatus and Systems 84: 338-343. 1965.

12. Faddeeva, V. N. Computational methods of linear algebra. Dover, New York, New York. 1959.
13. Friedlander, G. D. The Northeast power failure: a blanket of darkness. Institute of Electrical and Electronics Engineers Spectrum 3: 54-63. February 1966.
14. Heffron, W. C. and Phillips, R. A. Effect of modern amplidyne voltage regulator on under-excited operation of large turbine generators. American Institute of Electrical Engineers Transactions 71, Part 3: 692-697. 1952.
15. Imad, F. P. and VanNess, J. E. Fortran IV programs for finding eigenvalues by the QR transform. SHARE Computer Program SDA 3006-01. International Business Machines Corp., Poughkeepsie, New York. 1963.
16. Institute of Electrical and Electronics Engineers. Working Group of the Excitation Systems. Subcommittee of the Power Generating Committee. Computer representation of excitation systems. Unpublished paper presented at the Summer Power Meeting of the Institute of Electrical and Electronics Engineers, Portland, Oregon. Institute of Electrical and Electronics Engineers, New York, New York. 1967.
17. Kekela, J. and Firestone, L. Underexcited operation of generators. Institute of Electrical and Electronics Engineers Transactions on Power Apparatus and Systems 84: 811-817. 1964.
18. Kilgore, L. A. Effect of saturation on machine reactances. Electrical Engineering 54: 545-550. 1935.
19. Kimbark, E. W. Power system stability. Vols. 1 and 3. John Wiley and Sons, Inc., New York, New York. 1948.
20. Kron, G. Tensor analysis of networks. John Wiley and Sons, Inc., New York, New York. 1939.
21. Laughton, M. A. Matrix analysis of dynamical stability in synchronous multi-machine systems. Institution of Electrical Engineers Proceedings 113: 325-336. 1966.

22. Lokay, H. E. and Bolger, R. L. Effect of turbine-generator representation in system stability studies. Institute of Electrical and Electronics Engineers Power Apparatus and Systems Transactions on Power Apparatus and Systems 84: 933-942. 1965.
23. Olive, D. W. New techniques for the calculation of dynamic stability. Institute of Electrical and Electronics Engineers Transactions on Power Apparatus and Systems 85: 767-782. 1966.
24. Parks, R. H. Two-reaction theory of synchronous machines. Part 1. American Institute of Electrical Engineers Transactions 48: 716-730. 1929.
25. Parks, R. H. Two-reaction theory of synchronous machines. Part 2. American Institute of Electrical Engineers Transactions 52: 352-355. 1933.
26. Rankin, A. W. Per-unit impedance of synchronous machines. Part 1. American Institute of Electrical Engineers Transactions 64: 569-572. 1954.
27. Rankin, A. W. Per-unit impedance of synchronous machines. Part 2. American Institute of Electrical Engineers Transactions 64: 839-841. 1954.
28. Schleif, F. R. and White, J. H. Damping for Northwest-Southwest tieline oscillations: an analog study. Institute of Electrical and Electronics Engineers Transactions on Power Apparatus and Systems 85: 1239-1247. 1966.
29. Storry, J. O. Digital analysis of power systems using hybrid-parameters. Unpublished Ph.D. thesis. Library, Iowa State University, Ames, Iowa. 1967.
30. Undrill, J. M. Dynamical stability calculations for an arbitrary number of interconnected synchronous machines. Institute of Electrical and Electronics Engineers Transactions on Power and Apparatus Systems 87: 835-844. 1968.
31. VanNess, J. E. Root loci of load frequency control systems. Institute of Electrical and Electronics Engineers Conference on Power Industry Computer Applications Proceedings 1963: 3-23. 1963.

32. VanNess, J. E. and Boyle, J. M. Sensitivity of large multiloop control systems. Institute of Electrical and Electronics Engineers Transactions on Automatic Control 10: 308-315. 1965.
33. Westinghouse Electric Corporation. Electrical transmission and distribution reference book. Fourth edition. Westinghouse Electric Corporation, Pittsburgh, Pennsylvania. 1950.
34. Young, C. C. and Webler, R. M. A new stability program for predicting dynamic performance of electrical systems. American Power Conference Proceedings 1967: 1126-1138. 1967.

XI. ACKNOWLEDGMENTS

The author wishes to express his appreciation to his major professor, Dr. P. M. Anderson, for his suggestions and encouragement throughout the investigation. The author also wishes to thank Dr. R. J. Lambert for his help and advice on numerical analysis problems, and Mrs. Donald P. Smith (June Smith) for assistance in finding and using computer programs. Finally, the author wishes to thank Mr. G. L. Landgren of the Systems Planning Division of the Commonwealth Edison Company for data and his assistance which added practical and personal stimulus to this investigation.

XII. APPENDIX A, LIST OF PRINCIPAL SYMBOLS

(Second column is the Fortran program name)

Generator and Transmission Line

Values are in pu on the machine base unless otherwise noted. Direct axis is denoted by d. Quadrature axis is denoted by q.

p.f.	PF	Power factor
r.f.	RF	Reactive factor
	SF	Complex of (PF,RF)
V_b	VB	Magnitude of infinite bus voltage
	VBR	Real part of VB
	VBJ	Imaginary part of VB
	VBB	Complex of (VBR,VBJ)
V_t	VT	Magnitude of generator terminal voltage; also real part of VT (VT taken as reference axis)
Y_{sh}	YS	Half of adjusted shunt admittance of the trans- mission line
Z_L	ZL	complex adjusted impedance of transmission line
I_{sh}	AIS	Complex current through YS
I_b	AIB	Complex current through ZL
	EPA	Complex voltage behind Potier reactance
E_p	EP	Magnitude of EPA
x_p	XP	Potier reactance
	AMPP	Complex generator current with respect to VT

I	AMP	Armature current, (magnitude of AMPP)
r_e	RE	Magnitude of the real part of ZL
x_e	XE	Magnitude of the imaginary part of ZL
r_a	RARM	Resistance of armature
r_d	RD	Resistance of armature, d-axis
r_q	RQ	Resistance of armature, q-axis
x_d	XD	Reactance of generator, d-axis
x_q	XQ	Reactance of generator, q-axis
x'_d	XDD	Transient reactance of generator
R_d	RRD	$RD + RE$
R_q	RRQ	$RQ + RE$
X_d	XXD	$XD + XE$
X_q	XXQ	$XQ + XE$
P	AMW	Power output, mva x p.f.
D	D	System damping, including effect of damper circuits
H		Inertia constant, mws/mva
f		Frequency, 60 HZ
M	AM	$H/\pi f$
τ_f	TF	Open-circuit generator-field time constant, sec.
τ'_{dz}	TTDZ	Effective field time constant, sec.
ϕ	PHIR	Angle between V_t and I, rad.
δ_b	DELBR	Angle between V_t and V_b , rad.
δ	DELR	Angle between V_b and q-axis, rad.
δ_q	DELQR	Angle between V_t and q-axis, rad.
δ_t	DELTR	Angle between I and q-axis, rad.

δ_I	Angle between V_t and infinite-bus ref. axis
I_d	AMPD d-axis component of I
I_q	AMPQ q-axis component of I
V_d	VD d-axis component of V_t
V_q	VQ q-axis component of V_t
ψ_d	PSID d-axis component of flux-linkages
ψ_q	PSIQ q-axis component of flux-linkages
ψ_f	Field flux linkages; $\psi_{fd} = \psi_f x_{fd} / x_f$
$A_1 \dots A_1$	Machine coefficients related to the operating point
x_f	Reactance of field winding
x_{fd}	Mutual reactance between field and d-axis armature winding
i_f	Field current; $i_{fd} = i_f x_{fd}$
v_f	VF Field voltage
v_{fd}	EFD $V_f x_{fd} / r_f$
r_f	Resistance of field winding
T_m	Mechanical torque
T_e	Electrical torque
T_a	Accelerating torque
p	d/dt
λ	Eigenvalues
\underline{A}	Matrices

<u>M</u>	Matrices
<u>G</u>	Matrices
<u>F</u>	Matrices

Excitation System

K_A	AKA	Regulator gain
K_E	AKE	Exciter constant related to self-excited field
K_F	AKF	Regulator stabilizing circuit gain
S_E	SE	Exciter saturation function
T_A	TA	Regulator amplifier time constant, sec.
T_E	TE	Exciter time constant, sec.
T_F	TVF	Regulator stabilizing circuit time constant, sec.
T_R		Regulator input filter time constant, sec.
K_D	AKD	Power signal damping gain
T_D	TDA	Power signal damping time constant, sec.
V_R		Regulator output voltage
V_E	VE	Intermediate voltage point

Governor and Prime Mover

T_1	GT1	Control system time constant, sec.
T_2	GT2	Water hammer correction, sec.
T_3	GT3	Servo value motor time constant, sec.
T_4	GT4	Steam system time constant, sec.
T_5	GT5	Reheat steam system time constant, sec.
K_1	GK1	Overall gain (pu power/rad./sec.)
K_2	GK2	Fraction of steam reheated

XIII. APPENDIX B, SYSTEM EQUATIONS AND
A-CONSTANTS OF THE SYNCHRONOUS GENERATOR

A. General Non-linear System Equations

Using Park's notation (25,26) with direct and quadrature-axis windings, the equations relating machine fluxes and currents are

$$\begin{bmatrix} \psi_{fd} \\ \psi_d \\ \psi_q \end{bmatrix} = \begin{bmatrix} 1 & -(x_d - x'_d) & 0 \\ 1 & -x_d & 0 \\ 0 & 0 & -x_q \end{bmatrix} \cdot \begin{bmatrix} i_{fd} \\ i_d \\ i_q \end{bmatrix} \quad (54)$$

where

$$x'_d \simeq x_L = x_d - \frac{x_{fd}^2}{x_f}$$

$$i_{fd} \equiv x_{fd} i_f$$

$$\psi_{fd} = \frac{x_{fd}}{x_f} \psi_f$$

ψ_{fd} is the variable representing the per-unit field flux linkages dependent upon the field/armature mutual reactance and field leakage reactance ratio.

The following assumes that voltages induced in the armature do not vary from voltages induced at synchronous speed, and that the armature voltages induced by the rate

change of flux linkages during rotor oscillation are negligible compared to those generated by the synchronous rotating flux.

$$\begin{bmatrix} v_{fd} \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 1 + \tau_{fd} p & -\tau_{fd} p (x_d - x'_d) & 0 \\ 0 & -r_d & x_q \\ 1 & -x_d & -r_q \end{bmatrix} \cdot \begin{bmatrix} i_{fd} \\ i_d \\ i_q \end{bmatrix} \quad (55)$$

where

$$v_{fd} = \frac{x_{fd}}{r_f} v_f = (1 + \tau_{fd} p) i_{fd}$$

When the machine is considered to be connected to an infinite bus, proceeding from the internal machine conditions to that of the equivalent network, the following equations can be deduced from Figure 1.

$$\begin{aligned} v_d &= V_b \sin \delta + r_e i_d - x_e i_q \\ v_q &= V_b \cos \delta + x_e i_d + r_e i_q \end{aligned} \quad (56)$$

The restraints on the change of rotor position, due to machine inertia and system damping, are expressed as

$$M p^2 = T_a = T_m - T_e$$

or, more generally,

$$(M p^2 + D p) \delta = T_m - (\psi_d i_q - \psi_q i_d) \quad (57)$$

and the terminal conditions

$$\begin{aligned} V_t^2 &= v_d^2 + v_q^2 \\ I^2 &= i_d^2 + i_q^2 \end{aligned} \quad (58)$$

The above equations define the state of the machine with no control loops present.

B. A-constants

The A-constants derived in Equation 4 and given in Equation 5 are as follows from Laughton (21):

$$\begin{aligned} A_1 &= \left(\frac{V_b}{-R_d R_q - X_q X'_d} \right) \{ (\psi_q + i_q x'_d) (R_q \cos \delta - X_q \sin \delta) \\ &\quad + (\psi_d + i_d x_q) (R_d \sin \delta + X'_d \cos \delta) \} \end{aligned}$$

$$A_2 = \left(\frac{1}{-R_d R_q - X_q X'_d} \right) \{ \psi_d + i_d x_q R_d + i_q (R_d R_q + X_q x_e) - \psi_q X_q \}$$

$$A_3 = \left(\frac{1}{-R_d R_q - X_q X'_d} \right) \{ -V_b (x_d - x'_d) (R_q \cos \delta - X_q \sin \delta) \}$$

$$A_4 = \frac{R_d R_q + X_d X_q}{-R_d R_q - X_q X'_d}$$

$$\begin{aligned} A_5 &= \frac{1}{-R_d R_q - X_q X'_d} \frac{V_b}{V_t} [-v_d \{ (x_q X'_d + r_d R_q) \cos \delta + (r_e x_q - r'_d x_e) \sin \delta \} \\ &\quad - v_q \{ (r_e x'_d - x_e r_q) \cos \delta + (-x'_d X_q - r_q R_d) \sin \delta \}] \end{aligned}$$

$$A_6 = \left(\frac{1}{-R_d R_q - X_q X'_d} \right) \left\{ \frac{-v_d}{V_t} (r_e x_q - r_d x_e) - \frac{v_q}{V_t} (r_e R_d + x_e X_q) \right\}$$

$$A_7 = \left(\frac{1}{-R_d R_q - X_q X'_d} \right) V_b \left\{ \frac{-i_d}{I} (-R_q \cos \delta + X_q \sin \delta) \right.$$

$$\left. \frac{-i_q}{I} (X'_d \cos \delta + R_d \sin \delta) \right\}$$

$$A_8 = \frac{1}{-R_d R_q - X_q X'_d} \frac{-i_d}{I} X_q - \frac{i_q}{I} R_d$$

Also, the transient field time constant defined by

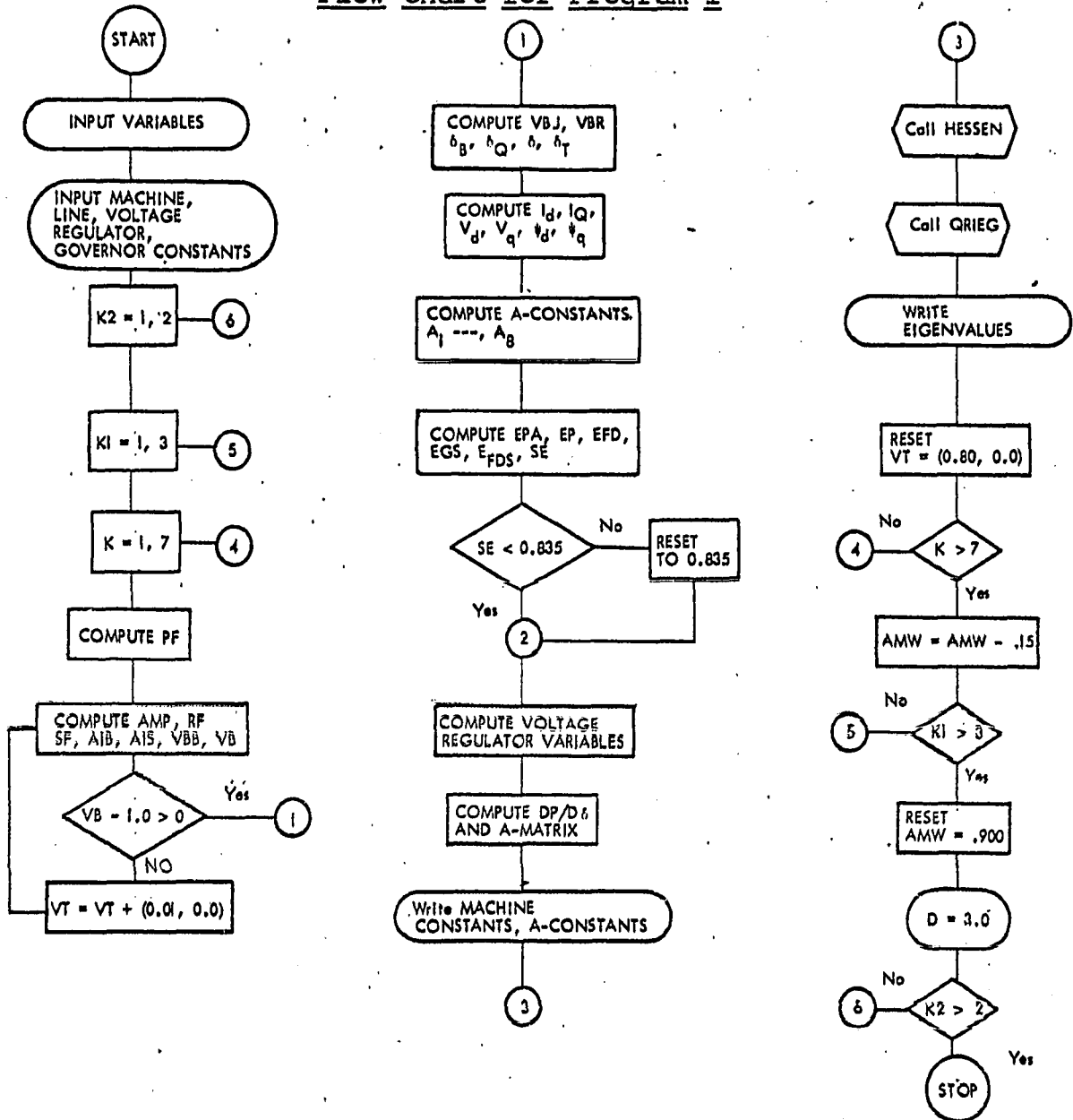
$$\tau'_{dz} = \left(\frac{R_d R_q + X_q X'_d}{R_d R_q + X_q X_d} \right) \tau_f$$

reduces the second row, second column diagonal element in Equation 4 to

$$-A_4 (1 + \tau'_{dz} p). \quad (59)$$

XIV. APPENDIX C, PROGRAMS FOR A-MATRICES AND
EIGENVALUES FROM COMPONENT DATA

Flow Chart for Program 1



```

C      PROGRAM 1
C
C      DETERMINING SYSTEM STABILITY BY EIGENVALUES.
C      WITH GOVERNOR, X-LINE=.414 PU, V-BUS=1.0 PU.
C      VOLTAGE REGULATOR SET AT NOMINAL VALUES.
C      POWER OUTPUT AT 1.0, .833 AND .667 PU.
C      MACHINE DAMPING AT D=2 AND D=3.
C      .90 PF LAG TO .95 LEAD IN .025 INCREMENTS.
C      FOR LAG PF, RF IS (-). FOR LEAD , RF IS (+).
C
C      DIMENSION A(30,30), ROOTR(30), ROOTI(30)
C      COMPLEX*8 VBB,VT,ZL,SF,YS,AIS,AIB,EPA,XP,AMPP
C      READ(1,990) ZL,VT,AMW,YS
990    FORMAT(7F10.4)
C      XD=1.918
C      XQ=1.896
C      XP=(0.,.35)
C      XDD=.333
C      RQ = .0018
C      RD = .0018
C      RARM=.0018
C
C      C1 START. INITIAL VALUES CORRESPONDING TO
C      X-LINE=.414 PU.
C      VT=(1.15, 0.)
C      ZL=(.027,.414)
C      YS=(0.,.062)
C      RE=.027
C      XE=.414
C      RRD=.029
C      RRQ=.029
C      XXD=2.332
C      XXQ=2.310
C      XXDD=.747
C      C1 END.
C
C      AMW=.9000
C      AM=.01367
C      D=2.0
C      TF=4.83
C      VOLTAGE REGULATOR CONSTANTS
C      AKE=1.
C      AKT=.10
C      TA=.02
C      TE=.51
C      TVF=1.
C
C      C2 START. VOLTAGE REGULATOR NOMINAL VALUES.
C      AKA=400.
C      AKF=.06
C      AKD=4.

```

```

      TDA=1.
C      C2 END.
C
C      GOVERNOR
      GT1=.15
      GT2=0.
      GT3=.05
      GT4=.10
      GT5=10.
      GK1=.0443
      GK2=.23
C
C      C3 START
      DO 22 K2=1,2,1
      DO 21 K1=1,3,1
C      C3 END.
C
C      C4 START.
      DO 11 K=1,7,1
      IF (K-4) 2,2,3
2      AI = K
      PF=.875+.025*AI
      PHIR = -ARCOS(PF)
      PHID = 180. * PHIR / 3.141593
      GO TO 4
3      AI = K
      PF=1.125-.025*AI
C      C4 END.
C
      PHIR = ARCOS(PF)
      PHID = 180. * PHIR / 3.141593
4      AMP=AMW/(PF*VT)
      RF = SIN (PHIR)
      SF = CMPLX(PF,RF)
      AIS=VT*YS
      AIB=AMP*SF-AIS
      VBB=VT-AIB*ZL
      VB =CABS(VBB)
      IF (VB-1.0) 13,10,10
13     VT=VT+ (.01,0.)
      GO TO 4
10     VBR=REAL(VBB)
      VBJ=AIMAG(VBB)
      AMPP=AMP*SF
      DELBR = ATAN2(VBJ,VBR)
      DELBD = 180. * DELBR/3.141593
      Y=AMP*XQ*PF+AMP*RARM*RF
      Z=CABS(VT)+AMP*RARM*PF-AMP*XQ*RF
      DELQR=ATAN2(Y,Z)
      DELQD = 180. * DELQR /3.141593
      DELR =-DELBR + DELQR

```

```

DELD =-DELBD + DELQD
DELTR  =-PHIR + DELQR
DELTD  = 180. * DELTR  /3.141593
AMPQ=AMP*COS(DELTR)
AMPD=AMP*SIN(DELTR)
VD=SIN(DELQR)*CABS(VT)
VQ=COS(DELQR)*CABS(VT)
PSIQ=-AMPQ*XQ
PSID=VQ+AMPQ*RARM
A1=(VB/(-RRD*RRQ-XXQ*XXDD))*((PSIQ+AMPQ*XDD)
1(RRD*COS(DELTR)-XXQ*SIN(DELTR))+(PSID+AMPD*XQ)
2*(RRD*SIN(DELTR)+XXDD*COS(DELTR)))
A2= 1./(-RRD*RRQ-XXQ*XXDD)*((PSID+AMPD*XQ)*
1RRD+AMPQ*(RRD*RRQ+XXQ*XE)-PSIQ*XXQ)
1(RRQ*COS(DELTR)-XXQ*SIN(DELTR)))
A3= 1./(-RRD*RRQ-XXQ*XXDD)*(-VB*(XD-XDD)*
A4=(RRD*RRQ+XXD*XXQ)/(-RRD*RRQ-XXQ*XXDD)
A5= 1./(-RRD*RRQ-XXQ*XXDD)* VB/VT *(-VD*((XQ
1*XXDD+RD*RRQ)*COS(DELTR)+(RE*RQ-RD*XE)*SIN
2(DELTR))-VQ*((RE*XDD-XE*RQ)*COS(DELTR)+(-XDD*
3XXQ-RQ*RRD)*SIN(DELTR)))
A6=1./(-RRD*RRQ-XXQ*XXDD)*(-VD/VT*(RE*RQ-RD*
1XE)-VQ/VT*(RE*RRD+XE*XXQ))
1(-RRQ*COS(DELTR)+XXQ*SIN(DELTR))-(AMPQ/AMP)*
A7=1./(-RRD*RRQ-XXQ*XXDD)*VB*((-AMPD/AMP)*
2(XXDD*COS(DELTR)+RRD*SIN(DELTR)))
A8=1./(-RRD*RRQ-XXQ*XXDD)*(-AMPD/AMP)*XXQ-
1(AMPQ/AMP)*RRD
TTDZ=(RRD*RRQ+XXQ*XXDD)*(TF)/(RRD*RRQ+XXQ*
1XXD)
E=PSID+XDD*AMPD
EPA=VT+AMP*SF*XP
EP=CABS(EPA)
EFD=VQ+AMPQ*RARM+AMPD*XD
EGS=.035*EXP(7.358*(EP-.8))
EFDS=EFD+EGS
SE=.235*EXP(.214*(EFDS-.8))
IF (SE-.835) 40,41,41
40 GO TO 42
41 SE=.835
42 CONTINUE
C VOLTAGE REGULATOR
AV1=(TE*(TA+TVF)+TA*TVF*(AKE+SE))/(TA*TE*
1TVF)
AV2=((AKE+SE)*(TA+TVF)+TE+AKA+AKF)/(TA*TE*
1TVF)
AV3=(AKE+SE)/(TA*TE*TVF)
BV2=AKA/(TA*TE)
BV3=AKA/(TA*TE*TVF)
D2LQR=2.*DELQR
DPD=(E*VT*COS(DELQR))/XDD+((XDD-XQ)*VT*VT*

```

```

1COS(D2LQR))/(XDD*XQ)
C GOVERNOR
GBP1=GK1*(GT1-GT2)/(GT1*(GT1-GT3))
GBP2=GK1*(GT3-GT2)/(GT3*(GT3-GT1))
M=11
DO 17 I=1,11
DO 17 J=1,11
17 A(I,J)=0.
A(1,2)=1.
A(2,1)=A1/AM
A(2,2)=-D/AM
A(2,3)=A2/AM
A(3,1)=-A3/(A4*TTDZ)
A(3,3)=-1./TTDZ
C VOLTAGE REGULATOR
A(3,4)=-1./(A4*TTDZ*274.)
A(4,4)=-AV1
A(4,5)=1.
A(5,4)=-AV2
A(5,6)=1.
A(5,7)=BV2
A(6,4)=-AV3
A(6,7)=BV3
A(7,1)=-A5/TDA
A(7,2)=(AKT*AKD*DPD)/TDA
A(7,3)=-A6/TDA
A(7,7)=-1./TDA
C GOVERNOR
A(8,8)=-1./GT4
A(8,10)=-GBP1
A(8,11)=-GBP2
A(9,9)=-1./GT5
A(9,10)=-GBP1
A(9,11)=-GBP2
A(10,2)=-1.
A(10,10)=-1./GT1
A(11,2)=-1.
A(11,11)=-1./GT3
C
C C5 START.
WRITE(3,991)PF,RF,VT,SF,AMPP,AIS,AIB
991 FORMAT(202,12F10.5)
WRITE(3,991)VBB,VB,VBR,VBJ,DELBQ,Y,Z,DELQD,
1DELD,DELTD,AMP
WRITE(3,991)AMPD,AMPQ,VD,VQ,PSID,PSIQ,YS,ZL
WRITE(3,991)EPA,EP,EFD,EGS,EFDS,SE,D2LQR,DPD,D,AMW
WRITE(3,991)A1,A2,A3,A4,A5,A6,A7,A8
WRITE(3,992)AV1,AV2,AV3,BV2,BV3
992 FORMAT(202,5F20.1)
C C5 END.

```

```
      WRITE(3,993) PF,TDA,AKD,AKF,AKA,VB,VT,AMP,  
1AMW,D,XE  
993 FORMAT(@0@,12F10.5)
```

C

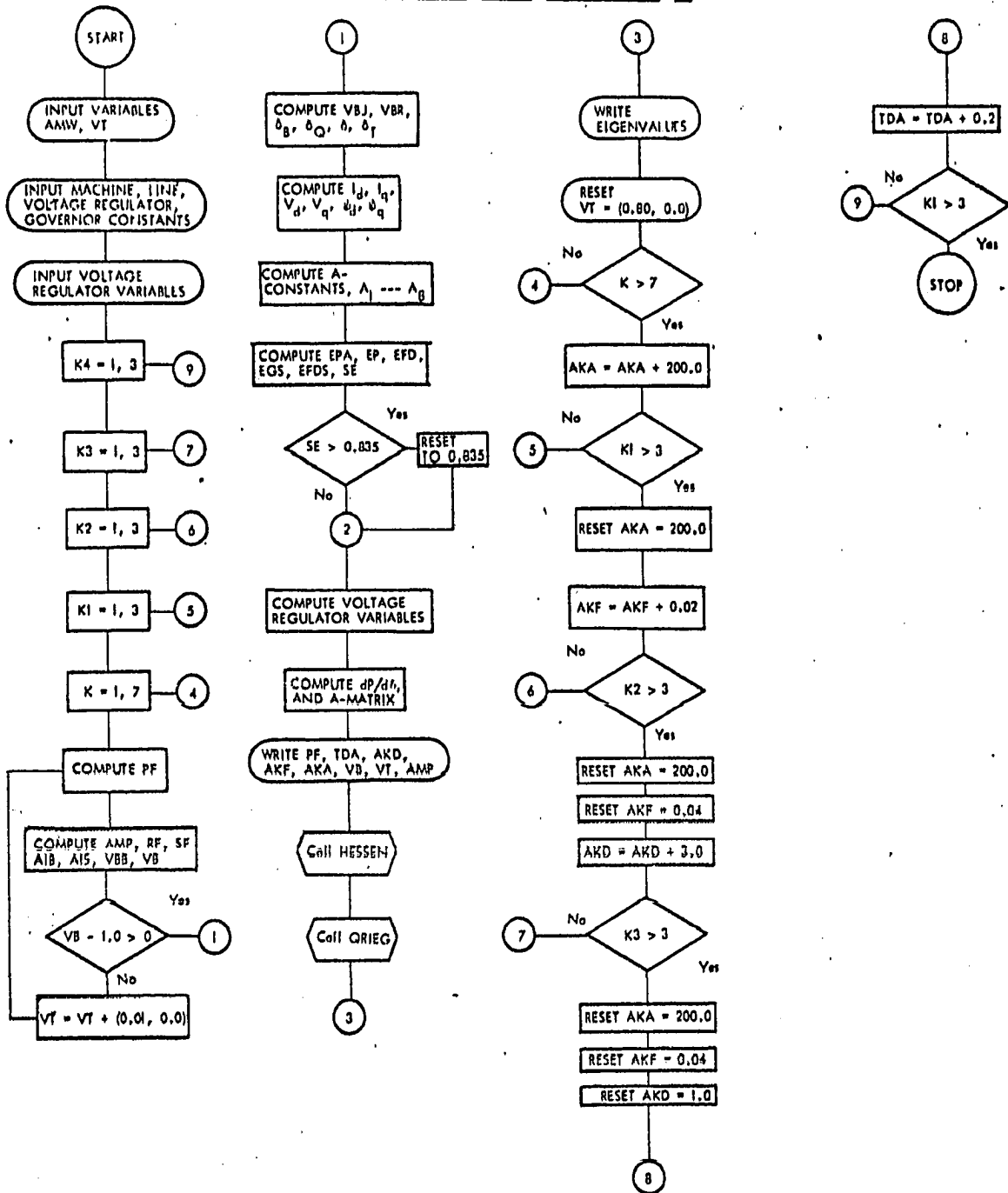
```
      CALL HESSEN (A,M)  
      IPRNT=0  
      CALL QREIG(A,M,ROOTR,ROOTI,IPRNT)  
      WRITE(3,8) (ROOTR(I), ROOTI(I), I=1,M)  
8      FORMAT(@ @,2F10.5)  
11     VT=(.80,0.)
```

C

C

```
      C6 START  
      AMW=AMW-.1500  
21     CONTINUE  
      AMW=.9000  
      D=3.  
22     CONTINUE  
C      C6 END.  
      END
```

```
C      PROGRAM 2
C
C      SAME AS PROGRAM 1 EXCEPT X-LINE = .718  PU.
C
C      REMOVE CARDS C1 START THROUGH C1 END.
C      REPLACE BY CARDS AS FOLLOWS.
C
C      C1 START.
C      VT=(1.25, 0. )
C      ZL=(.046,.718)
C      YS=(0.,0.106)
C      RE=.046
C      XE=.718
C      RRD=.0502
C      RRQ=.0502
C      XXD=2.636
C      XXQ=2.614
C      XXDD=1.051
C      C1 END.
C
C      END
```


Flow Chart for Program 3

```

C      PROGRAM 3
C      PROGRAM TO SCAN EIGENVALUES FOR STABILITY.
C      WITH GOVERNOR, D=2, X-LINE=.718 PU.
C      POWER OUTPUT=1.0 PU, V-BUS=1.0 PU.
C
C      VOLTAGE REGULATOR VARIED IN INCREMENTS AS
C      FOLLOWS.
C          TDA=.8, 1.0, 1.2
C          AKD=1.0, 4.0, 7.0
C          AKF=.04, .06, .08
C          AKA=200., 400., 600.
C
C      REMOVE CARDS C2 START THROUGH C2 END.
C      REPLACE BY CARDS AS FOLLOWS.
C
C      C2 START.
C      VOLTAGE REGULATOR VARIABLES INITIALIZATIONS
C          TDA=.8
C          AKD=1.0
C          AKF=.04
C          AKA=200.
C      C2 END
C
C      REMOVE CARDS C3 START THROUGH C3 END.
C      REPLACE BY CARDS AS FOLLOWS.
C
C      C3 START
C          DO 24 K4=1,3,1
C          DO 23 K3=1,3,1
C          DO 22 K2=1,3,1
C          DO 21 K1=1,3,1
C      C3 END.
C
C      REMOVE CARDS C4 START THROUGH C4 END.
C      (THIS DATA HAS BEEN FOUND IN PROGRAMS 1+2).
C      REMOVE CARDS C5 START THROUGH C5 END.
C      REPLACE BY CARDS AS FOLLOWS.
C
C      C5 START.
C          21 AKA=AKA+200.
C             AKA=200.
C          22 AKF=AKF+.02
C             AKA=200.
C             AKF=.04
C          23 AKD=AKD+3.
C             AKA=200.
C             AKF=.04
C             AKD=1.0
C          24 TDA=TDA+.2
C      C5 END.
C      END

```

```
C      PROGRAM 4
C      PROGRAM TO OBTAIN .005 INCREMENT SCANNING OF
C      POWER FACTORS THROUGH .98 LAG PF TO 1.00 .
C
C      REMOVE CARDS C4 START THROUGH C4 END.
C      REPLACE BY CARDS AS FOLLOWS.
C
C      C4 START
C      DO 11 K=1,5,1
C      IF (K-4) 2,2,3
C      2 AI=K
C      PF=.975+.005*AI
C      PHIR=-ARCOS(PF)
C      PHID=180. * PHIR/ 3.141593
C      GO TO 4
C      3 AI=K
C      PF=1.025-.005*AI
C      C4 END.
C
C      END
```